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IN THE
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A Study of Differentiable Mental Abilities

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PREFACE

In the first chapter of this book I endeavor to establish that a method of proving the independence of two or more mental traits or capacities lies at the root of a comprehensive study of mental organization. My connection with the development of a technique to accomplish this end extends back in time more than a decade. The problem readily grew out of an interest in guidance and in the inter-correlation of mental abilities. The work of Thorndike along these lines was an early and potent influence, while the early and late work of Spearman has always been most intimate in its essential purpose and in its techniques.

Incidentally not a few of the findings of my study, which it was thought would be new, are foreshadowed or specifically cited in Spearman's last work, *The Abilities of Man* (1927). This is really most fortunate, for it lays the foundation for future work with the promise of fruitful outcome not possible without such corroboration. The experimental determination of mental types would provide a basis for psychology which unfortunately is now quite lacking. The essential requisite is a technique for testing the agreement of any postulation of independent mental traits with observed facts. In my judgment, the chief claim to merit of the present study lies in its bearing upon procedure even more than in its specific findings, however intimate these may be with the immediate problems of guidance and individual differences.

I am indebted to Dr. E. P. Cubberley, Dean of the School of Education with which I am connected, for encouragement and facilities for the conduct of this study, and to the Commonwealth Fund for general financial assistance and for a fellowship, which was very ably filled by Mr. C. R. Brolyer. Chapter ix is a study of data found in published sources. As there noted, various students have been responsible for carrying through the statistical and interpretative work leading to certain of these analyses.

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CHAPTER I

THE BOUNDARIES OF MENTAL LIFE AND A TECHNIQUE FOR THEIR INVESTIGATION

The study of the nature and scope of mental traits is as broad as the entire field of psychology, and just about as evasive as the all-pervasive ether. He who dedicates his services to "human welfare" is rather less likely to ease the progress of mankind than, say, an ear doctor who limits his attention to a narrower field. Nevertheless it is necessary to envisage an entire realm in order to maintain a proper perspective, to keep poised in the turmoil of separate schools, neo-schools, and counter-schools. Each separate school is generally willing to ignore the others with the fine tolerance of the wise toward the harmlessly demented. If perchance a behaviorist takes issue with a psychoanalyst, the latter blandly informs him that he does not know the facts and the principles of the latter's study, and in this he is probably right. How, then, as each school becomes more specialized in technique, more confined within its self-made walls, and more dependent upon the rapidly increasing fruits of its own garden, is there any chance for helpful criticism of school by school, or any prospect of co-ordination of their several doctrines? Not only is there prospect, but sooner or later co-ordination is inevitable, for the doctrines meet at the crossroads of life. Professor Blank found bluejays breaking the eggs in a quail's nest, and killed seven in as many shots. He was "so mad that he couldn't miss." What a fine interplay of traits! Usually emotions and motor co-ordinations run separate courses, but here they met at the quail's nest.

It would seem that just two things are necessary to a comparative and co-ordinative study: First, a technique that is universal in its applicability in the sense that it serves not only at the crossroads, but along the remote stretches where no other highway is near. And, second, the will and the opportunity on the part of someone to apply this technique to the divergent and convergent paths of modern psychology. The technique that is necessary is to be merely deductive, for its object is to test the validity of claims made and supported by sundry schools, and it is not primarily interested in the way these various claims happen to have been conceived in the first instance.

This single method of valuation applies to all schools of psychology, because in one important regard they all do the same thing, in that each advances certain psychological elements as underlying its particular system—the behaviorist has his original tendencies and "given" nervous structure, the gestalt psychologist his configurations and *e pluribus unum* doctrine, the Freudian his underlying sexual urges, and so on. Without expressly so stating, the very postulation of these different elements constitutes a claim that each is an entity in itself and is entitled to an independent status in the field of mental life. Thus all that is needed in the way of technique by him who would investigate the entire field is a device for testing the independence of any given element from all others or at least from such as may be thought to be somewhat similar to it.

Before such testing there must be a definition of the element, and the school of psychology in question is obligated to supply this. This definition must be invariable, and expressible in terms of conduct. Certainly it would be in reason to demand as much in any other field. If a certain doctor maintains that hives are caused in one person by tomatoes, in some other individual by any sort of protein, and in a third person by some particular protein, unless caused by the lack of some protein, his very comprehensive formulation is not subject to experimental test. About all that can be done with such a formulation, whether in psychology, medicine, astrology, economics, or any other department of human activity is to let it alone and see that it does not intrude upon more promising hypotheses.

The multiplicity of causes put forward by certain psychoanalysts for a single outcome nearly precludes the subjection of their claims to scientific inquiry. If it is claimed that one performance is consequent to one certain capacity, the matter may be readily tested. Even if the performance is a compound of two capacities check-up is possible. As the number of possible causes increases to three, four, or more, the possibility of check-up rapidly vanishes, and science goes through pseudo-science to speculation or to charlatanry. The trouble is not primarily in postulating several contributing causes but in doing this antecedent to the development of a method of proof. In any true science the formulation and the means of at least partial verification, generally by noting necessary objective consequences, run hand in hand.

Thus, in the field of psychology, if a designation of some trait or capacity, as a category of mental life, is to be given serious consideration, it must be such as to reveal itself as a measurable difference in conduct, that is, as a measurable difference in the same individual at different times, or in different individuals at the same time. Does a trait like introversion meet these conditions? The number of different verbal statements of the meaning to be attached to this term falls but a little short of the number of people using it in writing. Many of these meanings are so subjective as to lead one to doubt whether differences of conduct can be related to them. No method of verification can be hoped for or in fact desired that will investigate the reality of so indefinite a concept. If one or more of the users of the term state that they mean thereby high scoring on a designated test which is definite, administered in a standardized manner, and which yields an objective score, then, and practically only then, can the matter be subjected to test.

This demand that a concept be subjected to objective measurement before it is worthy of serious consideration as an independent category of mental life, though sweeping, is not too sweeping, if we limit objective measurements to such as are definable and verifiable. How about the large class of concepts which are definable only in a rough way and verifiable only in part? Suppose we define "honesty" in the following manner: "It is a trait possessed in varying amounts by school children and recognized by teachers, with the result that when teachers rank their pupils on the basis of honesty, a measure of the trait is obtained." Let us first note that in this statement no precise distinction has been made between honesty itself and the measure of it. To determine whether such a lack of distinction is justified we should attempt to verify the measure. If a second equally trustworthy teacher having equal familiarity with certain pupils gives a rank order which is the same as a first teacher's, and if a third teacher, a fourth teacher, etc., all do the same, then the measure is verified and there is no need of distinction between the trait and the measure of it. Such a situation would arise in practice if height instead of honesty were the trait in question. We say, "John is four feet eight inches tall," and do not quibble over the fact that "four feet eight inches" is merely some person's measurement of John. If there is complete agreement the measurement is the trait for all practical purposes. In the case of honesty there would be no complete agreement but partial agreement only. Does such a measure provide a basis for scientific investigation? It seems to the writer that it does, provided (1) the degree of agreement of a measure in hand with a second equally trustworthy measure is known, (2) the technique adopted takes the unreliability of the measure into account and allows for it so that no systematic error is introduced, and (3) the technique adopted guards, by drawing tentative conclusions where necessary, against any chance error which may be introduced due to this unreliability of the measure.

These are necessary qualifications, but when these precautions are taken it would seem that objective measures in the sense earlier insisted upon may be derived from sources no more specific than the judgments of acquaintances. Though this is true, it is surely the part of wisdom to utilize performance records which are independent, or nearly independent, of human judgments whenever possible and when dealing with the more far-reaching issues.

Some very suggestive studies of mental capacity based upon judgments have been made. The following may be mentioned as of special value: Shen, "The Validity of Self-Estimate" (1925), "The Reliability and Correlation of Personal Ratings on Certain Traits" (1924); Webb, "Character and Intelligence" (1915);

Magson, "How We Judge Intelligence" (1926) ; Thorndike, Bregman, and Cobb, "The Selection of Tasks of Equal Difficulty by a Consensus of Opinion" (1924) ; and Thorndike, "A Constant Error in Psychological Ratings" (1920).[1] After exercising all possible care there remain ambiguities in the interpretation of judgment measures. Should ten, twenty, or even one hundred acquaintances give judgments, which, pooled, characterize a person as one standard deviation above the average for his age, in honesty, still no one knows just what is meant by them, and probably never can know, for he has no way of discovering how his independent concept agrees or disagrees with the average concept of these judges. On the other hand, if honesty is defined as a trait tending toward higher scores on a designated test, when administered and scored as directed, then any particular student can study the relationships of this "honesty," even though he might individually misinterpret the purport of the test questions.

A few years ago Garnett (1919) made an analysis of Webb's (1915) study based upon judgments, and concluded that there is a mental factor "cleverness." Certainly to understand this, "cleverness" is to be interpreted as by Garnett, but this we can only approximate. And further, Garnett deduced his "cleverness" from "quickness," "profoundness," "common sense," "originality," etc. (of 48 mental traits investigated, 43 came from judgments), which were traits appraised by Webb's judges. The meaning of these traits is certainly to be interpreted as by the average of Webb's judges, but as to these meanings both we and Garnett can secure only rough approximations, so that our final belief in "cleverness" as a factor must be most uncertain. Still more serious is the fact that it is incapable of verification, for we cannot duplicate by reproduction of the investigation, step by step, our own interpretation of Garnett's interpretation of Webb's judges' interpretation of the traits of the unknown subjects. All these difficulties except the

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[1] For bibliographical details, see Bibliography following chapter x below. References will hereafter be given to that list, by author, year, and, when necessary, key word.

last—differences in supposedly similar populations—are avoided when a trait is defined in terms of an objective measure which is capable of being duplicated and thus verified.

Because of the uncertainty of outcome when dealing with judgments in a study, the purpose of which is to determine disparate mental rubrics, this type of psychological investigation is only incidentally investigated in connection with the delightful study by Shen (see chapter x below). It is probably an important and legitimate field in the matter of discovery and preliminary survey, but it is truly of doubtful value in the subsequent steps of proof.

Definition and reproducibility are essential characteristics of the psychological data to be dealt with. Having data with these characteristics, and having purposes as unified as is the case with investigators, there is provided a situation which is amenable to a single type of analysis. The avowed purposes of psychologists sound quite different, but this need not deter us from approaching all investigations with a single comparative technique, because there is a fundamental underlying unity in these purposes. The determination of some difference is the object of every psychological formulation. For convenience these differences may be characterized as of three types, or of some composite of the three. Thus the difference may be chiefly affected by a change in time, as in the case of growth and fatigue, chiefly affected by spatial consideration, as are all of our sensory and motor functions, or chiefly related to differences in type of central nervous activity, as are the emotions and the several intellectual modes of thought, reasoning, memorizing, etc.

The technique of Spearman based upon tetrad differences (see page 47), for determining the discreteness of mental phenomena, and that used by the writer, involving much of Spearman's technique but also a considerable extension of it, is entirely adequate to determine whether two things, say a set of visual and a set of auditory measures, are basically the same or different. The case cited is so simple that its solution seems obvious. Much simpler devices than tetrad differences are available here, as perhaps in the case of all differences affected by spatial considerations, which enable one to establish that the auditory and visual phenomena are disparate. It would be ridiculous to measure both the visual and the auditory acuity of a large number of people, measure still other traits, and then calculate a lot of tetrad differences to see if they would warrant the conclusion that visual and auditory acuities are different things. Also, dealing with a single individual, it would be absurd to measure the sensitivity to light, to sound, to heat, etc., of different areas of the body surface, calculate tetrad differences and conclude that visual and the other sensitivities are not the same. However, this could be done, and by this technique each measurable sensory and motor trait of mankind that does stand alone could be so proved. We thus see that the technique in question is much broader in its applicability than one might judge from the field to which it has been applied. However ponderous the tetrad-difference method is in the case of these sensory and motor traits, it seems to be the only way, other than that of introspection, available in connection with purely mental phenomena. Apparently the reason is that the usual concomitants of sensory and motor traits, namely, space and time, are not conditions of mental life. One's reasoning power has not been localized in any particular end organ, and it is approximately timeless, operating with equal facility yesterday, today, and tomorrow. Mental phenomena are not entirely devoid of temporal characteristics, as will be discussed at greater length in connection with growth, but in so far as they are independent of it they are not amenable to study by the simple procedures which suffice with sensory and motor traits.

We may characterize the entire realm of psychological thought by saying that it is concerned (*a*) with differentiating between traits, or more broadly, though without a change in meaning, with studying the relationship between traits; (*b*) with determining changes in single traits (growth, fatigue) as time changes; and (*c*) with modification as locus (end-organ) changes. From a different point of view, the first of these is to be recognized as a study of individual differences. Of these three problems of difference, we are here concerned only with the first, for a trait must be determined as independent before it is very useful to attempt to determine its growth, if it does grow, or its locale, if it has one. Further, as to differentiating between traits we are here concerned only with those the independent status of which is open to question —memory, analysis, persistence, etc., but not visual acuity, right-hand grip, etc.

Though our field is thus limited, it may not be amiss to attempt to picture the structure of a human organism which is possessed of traits as wide apart as those of mankind. Dr. John F. Walker found (1924) that there are many independent motor and sensory capacities in children, thus suggesting specific independently conditioned origins. Professor Spearman (1927, *Abilities,* p. 217) reports much the same thing. Accordingly, so far as these things are concerned we may look to a rich and varied original genetic structure. The writer has argued elsewhere (1926, *Influence;* 1926, "Oddities") that this many-particled original nature is limited in its development by environmental pressure. If there are 40 chromosones, each with 1,000 genetic elements, and if there are five allelomorphic alternates for each element, then the number of permutations conceivable is $5^{40 \times 1000}$. The earth could thus be populated millions of times without duplicating in genetic structure any individual. However, probably many of these permutations are lethal, for the embryo in the main develops in one certain milieu only. Certainly medical science indicates a wonderful similarity in metabolism and tolerance, both in infants and in older people. If we carry this thought a step farther we can well believe that in the case of traits concerned with social contacts (love, hate, mental agility at argument, intellectual keenness in finding one's way, subtlety in conflict, tolerance toward the views of associates, etc.), certain permutations only are favorable to survival. Perhaps through generations the allelomorphic elements in these surviving combinations have become linked through mechanisms not unlike those of symbiosis. Thus genetic origins almost infinitely extensive, interacting with physiological and social environment, find their fruition in beings running more or less true to survival types in essential mental and physical make-up, but of great variability in nonessential physiological detail. To illustrate, we might readily conceive of almost an infinite number of intergrades in the matter of sex; but conditions of survival have determined that most of these must vanish, so that as a final result male and female children are born and few hermaphrodites. This picture[2] of genetic structure, though merely a picture, seems adequate to a portrayal of all the conditions of inter-relationship found, while it is at the same time not sufficiently specific to prejudice any of the experimental treatment. It has two important features in addition to those mentioned, in that it suggests in what evolution consists and it gives a definite place to both heredity and environment in this process.

We have noted that mental traits which we wish to study with a view to differentiating between them are of the class not readily localized in space and time. Even so the field is very extensive and includes what are commonly called the higher mental processes. Our problem is difficult because of the complexity of these processes and because of their number, for they cannot be studied singly. A single mental trait can be studied with reference to growth without any attention to other traits, but obviously when

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[2] The conclusions of Stewart in his "Mendelism in Bacteriology" (1926), as reported by E. F. Symmes (*Psychological Abstracts*, Vol. 1, No. 5, May 1927, Item No. 1111), are very pertinent in this connection: "The variations exhibited occur only in adaptation to the stimulus presented, this fact establishing an important hypothesis for evolutionary theory—that a race can adapt itself by variation directly to its environment and that such an adaptation is rigorously inherited. Such an acquired character, however, is inherited only by unicellular organisms; in higher forms it doubtless would affect the soma only, and not the germ-plasm, and therefore not be inherited." This is quoted because so far as unicellular organisms are concerned it very fully supports the hypothesis here mentioned. So far as higher forms of life are involved it need not be considered a reversal of Mendelian doctrine to postulate some functional relationship between environmental influences and linkage of the genetic elements possessed by surviving individuals, thus in fact giving hereditary conditions operating, though much more slowly, as in the case of the bacteria studied by Stewart.

the issue is the difference between traits, it must be studied in conjunction with other traits.

It is here argued that such an approach should logically precede all detailed studies of a trait. Professor Spearman argues that "perseveration," as measured by G. E. Müller, is the same as "introversion," as proposed by Jung. Surely so vital an issue as this could well be made a matter of first importance, even if the investigator is studying merely the one or the other trait, and not primarily trying to see things in proper perspective. If this latter is the object, then not only should introversion and perseveration be studied together, but also along with many other traits, of which some will probably be related to these two and some will be entirely independent. A comprehensive study of this last sort has never been made. The experimental work of the present volume has dealt jointly with general ability (probably, in the main, maturity), manipulation of spatial relationships, facility with numbers, facility with verbal material, memory, mental speed, and certain other less clearly defined traits. Professor Spearman and his students have studied these same abilities, except facility with verbal material as here defined, and other traits, but his studies have not been of all the traits at once. Generally speaking they have been investigations of the independence of each of the traits singly, from Spearman's general factor "*g*." These studies have been most fruitful in indicating real differences between certain traits and certain *g*'s (the writer does not quite subscribe to the view that the thing called "*g*" in all of these investigations is the same throughout), and in suggesting differences between these; but the much-needed comprehensive examination of relationships between many differently labeled, derived, measured, and variously sponsored mental traits is still to be made. Certainly such an investigation should take into consideration the following factors: (*a*) maturity; (*b*) sex; (*c*) race; (*d*) manipulation of spatial relationships in so far as independent of differences in visual acuity, etc.; (*e*) manipulation of auditory relationships in so far as independent of auditory acuity, etc.; (*f*) verbal facility; (*g*) number facility; (*h*) memory—one or more kinds; (*i*) mental speed; (j) one or more traits involving general motor organization and skill; (k) purpose or purposes; (l) ebullience or cleverness; (m) perseveration or intro-extraversion; (n) oscillation or variability in performance; (o) one or more traits connected with social interest and activities; (p) any remaining general factor not included in the preceding. The only adequate attack is to study all of these at once upon the same population, as otherwise certain relationships, perhaps of great importance, will be missed. Owing to previous work, mainly that of Spearman, the list as drawn up is very select and very promising for future study. The writer believes that the techniques herein employed are adequate for such a study and rather more comprehensive than those of Spearman, in spite of the fact that Spearman's have already yielded rich return.

In the study of independent mental capacities, Spearman has in the past utilized quite a number of different criteria. Though he claims that these early criteria are sound, because in harmony with results based upon what he now considers his final technique, there is considerable room for argument. If Technique B is sound and leads to a certain conclusion, then it does not follow that Technique A is sound because in some given instance it leads to the same conclusion. We are, however, not concerned at this time with Spearman's earlier techniques. His last one, for which he claims finality and universality, must be carefully scrutinized.

It is readily shown that if four variables have one, and only one, common factor running through them (in, the writer would add, a linear manner) then every tetrad difference involving the correlation coefficients of these four variables will be equal to zero. In the notation of this text, tetrad differences are denoted by the letter t with appropriate subscripts, and are defined as follows:

$$\left.\begin{aligned} t_{1\,2\,3\,4} &= r_{12}\,r_{34} - r_{13}\,r_{24} \\ t_{1\,2\,4\,3} &= r_{12}\,r_{34} - r_{14}\,r_{23} \\ t_{1\,3\,4\,2} &= r_{13}\,r_{24} - r_{14}\,r_{23} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\, [1]$$

The equality of these tetrads to zero in the case when one general factor is sufficient was first pointed out by Spearman. It is very easily proved in a number of ways, one of which is given in chapter iii.

If there are more than four variables, if one general factor only runs through the variables, and if they contain no group factor, i.e., a factor found in a number of the variables but not all of them, then every tetrad will equal zero. The converse of this is also readily proved, namely, if every tetrad does equal zero, then the variables may be thought of as having one general factor and no group factors. With this as a starting-point Spearman argues that if the distribution of obtained tetrad differences shows a variability no greater than would be expected as a matter of chance, then one and only one factor, other than factors specific to the separate variables, may be looked for in the several measures. To carry this argument into effect the distribution of tetrads must be made (based on all the possible tetrads resulting from the variables employed), and its standard deviation must be calculated and compared with the theoretical standard deviation in case all tetrads deviate from zero merely as a matter of chance. This line of reasoning and the execution of it must be examined very critically, for it constitutes Spearman's major technique and, according to him, the only technique which is adequate. First as to the execution of it: The formula (Spearman, 1927, Formula 16*a*, p. xi) which Spearman and Holzinger (1926) have derived for the standard deviation of the distribution of a population of tetrad differences is

$$\sigma = 2\left\{\frac{r^2(1-r)^2 + (1-R)s^2}{N}\right\}^{\frac{1}{2}} \qquad [2]$$

wherein r is the mean of all the r's and s^2 is their variance, and

$$R = 3r\frac{n-4}{n-2} - 2r^2\frac{n-6}{n-2}$$

In these equations n is the number of variables and N is the size of the population.

The writer has criticized the earlier formula (Spearman and Holzinger, 1926, Formula 1), now replaced by this one, and of the use of this later formula Spearman writes: ". . . . although on some theoretical points still awaiting further elucidation in practice at any rate [it] appears to be far more convenient, and even more reliable than [Spearman and Holzinger's formulas giving the probable errors of single tetrads]."[3] The proof of this very critical Formula 2 has not as yet appeared in print.

Formula 2 may be called not merely a critical formula, but *the* critical formula. Professor Spearman states (1927, p. 74): ". . . . whenever the tetrad equation holds throughout any table of correlations, and *only* when it does so, then every individual measurement of every ability can be divided into two independent parts . . . . [a general and a specific factor]." Formula 2 is the final criterion that Spearman uses to determine if the tetrads throughout the entire table are merely chance deviations from zero. He writes (1927, p. 137): "To begin with, a note of warning must be sounded against all attempts to replace the rigorously demonstrated criterion by anything else. Many writers have tried to invent a new one for themselves; others have declared that so many are in the field as to produce a difficulty in choosing between them. Against this, we must formally declare that no other rigorous criterion than that demonstrated here (including mere equivalent conversions of it) has ever been proved *or ever can be.*" Professor Spearman is presumably referring to the tetrad difference formula, $r_{12}\, r_{34} - r_{13}\, r_{24} = 0$, but we must note that the crux of the matter is not in the proposition that all tetrad differences equal zero, but in the standard errors of their actual values, which deviate by chance or otherwise from zero. According to Spearman's method this leads back to Formula 2. Clearly it is the basic formula in the entire treatment. If we grant that Formula 2 is correct, we may still question the use made of it, for Spearman assumes that chance would yield a normal distribution of tetrad differences with this standard deviation. This assumption of a normal distribution, even in situations where one general factor only is present does not seem reasonable, for the chance errors in the correlation coefficients are known to be correlated, so that we

[3] Spearman, 1927, Appendix, p. xi.

may expect the chance errors in the tetrads also to be correlated, and to an appreciable extent, for they are only functions of four correlation coefficients and the products of correlation coefficients in pairs are repeated many times in the total population of tetrad differences. This would yield a non-normal distribution, of just what form the writer does not know. The obvious way of using the chance standard deviation of a population of tetrad differences would be to compare it with the obtained standard deviation, find the difference between the two, and the standard error of this difference. This standard error, which will certainly be very small, is not known, and it probably cannot be determined by any means sufficiently simple to be serviceable. Should one object to this proposed method of interpretation on grounds similar to those just raised, namely, that the distribution of differences between these two standard deviations will not be normal and thus not readily interpretable, it can be shown that the point, though not without foundation, is much less material here than in the former case.

How material this particular criticism of Spearman's use of Formula 2 is, is not known. It may be quite trivial. A more important criticism from the writer's point of view is the fact that the situations in which one is really interested are not those to which it can usefully be applied. According to Spearman's latest conclusions there are no less than three general cognitive factors—*g*, oscillation, and perseveration—and a much larger number of group factors, including memory, a spatial factor, a conjunction factor (probably identical with what the present writer calls a number factor), a music factor, etc., as well as "conative" factors. All that Spearman's criterion could tell us would be that one factor was or was not sufficient to explain a given situation. All of the writer's data do show, and he ventures to prophesy that all of the forthcoming as well as much of the earlier data from Spearman's laboratory will show, the need for more than a single factor; thus Spearman's tool proves inadequate. We are no longer concerned with the first step, "Does one factor suffice?" but with the later steps, "How many and what factors suffice?" The writer presents herein (chapter iii) the complete solution of the adequacy of two factors in the case of variables up to the number five, but he has found even this to be inadequate, for there are found in his data more than two general or group factors. Finally, the writer presents an iteration method for handling the problem when more factors and more variables are present (chapter iii). This method seems to be a powerful analytical device. It calls, however, for a wider use and more detailed mathematical scrutiny than it has as yet received. As to the needed mathematical scrutiny, it can, in brief, be said that the writer has been using an iteration method and obtaining a convergency in a series without having first proved that convergency is a mathematical necessity.

The fundamental technique of Spearman has been supplemented by him by a partial correlation procedure. When he has found that one independent factor was insufficient, he has partialed out g and by a study of the residual correlations has found additional factors. This procedure depends upon the ability to obtain a measure of g uncontaminated by other factors. The difficulty of doing this is great, because of the special hazards involved in partialing out just what is needed and no more when dealing with measures having large chance elements in them, and perhaps also having disconcerting specific and group factors in them. The writer has not attempted to follow carefully all of this partial correlation treatment, because Spearman's populations have been very small, the partial correlations very small, and the resulting probable errors very large and, most unfortunately, unknown. He would, however, express the belief that in spite of these special hazards the method has much to commend it.

Though certain shortcomings in the tools used by Spearman have been pointed out, nevertheless the writer believes that on the whole he has used them with rare judgment and has determined the existence of many important mental factors. In short, as Spearman's technique and point of view with reference to the significance of differences (see chapter ix below, p. 215) seems too rigorous, or, in other words, unfavorable to the discovery of factors in addition to g, we may place rather special confidence in the reality of such special factors as he does report. The very thorough review of these to be found in his *Abilities of Man* (1927) makes it unnecessary to give more than a brief discussion of them here. Professor Spearman clearly distinguishes cognitive, conative, and affective traits, as well as various other traits, for example, retentivity and fatigue. It does not seem to the writer that he has established by his own method of tetrad differences the independence of these things. We will therefore, even at the risk of not doing full justice to his view, make no attempt to preserve his classification of traits and capacities. We will list those that he has found disparate, but we should bear in mind that in the main, by his technique, differences of each from *g* are established rather than complete difference of each from each other.

First in this list is Spearman's *g*. As measures of it, which involve nothing else except specific factors, are the "usual sets of mental tests" (1927, *Abilities,* p. 161). The number of specific tests which could here be mentioned is very great, including in addition to many others, opposites, synonyms, classification, completion, questions, analogies, paragraphs, meanings, memory, abstract thought, accuracy, inferences and likelihood (1927, pp. 224–27). Owing to the universality of *g,* Spearman states that "any test will do just as well as any other, provided only that its correlation with *g* is equally high" (1927, p. 197). The saturation of measures with *g* is not markedly affected by any differences in the fields of cognition (1927, pp. 203–4). Again, "*g* proved to be a factor which enters into the measurements of ability of all kinds, and which is throughout constant for any individual, although varying greatly for different individuals. It showed itself to be involved invariably and exclusively in all operations of eductive nature, whatever might be the class of relation or the sort of fundaments at issue. It was found to be equally concerned with each of two general dimensions of ability, Clearness and Speed. It also applied in similar manner to both the dimensions of span, which are Intensity and Extensity. But it revealed a surprisingly complete independence of all manifestations of Retentivity. Whether there is any advantage in attaching to this *g* the old mishandled label of 'intelligence' seems at least dubious" (1927, p. 411). In brief, Spearman's concept is that g is the ability to deduce relations and correlates, and is dependent upon a central fund of energy.

The experimental results reported in later chapters hardly support this concept, because there seem to be two, and perhaps three, traits combined in this one concept. First, there is a factor making for correlation between variables due to maturity, race, sex differences, and differences of antecedent nurture. That these things would strongly tend to introduce a general factor is shown in chapter ii of this book, and that Spearman pays far too little attention to them is very obvious to one going over the various experimental investigations that have been made in his laboratory and under his direction. In fact, he writes, "Also worthy of mention, though hardly of prolonged examination, is the taking of g to have reference only to children, being in fact no more than a measurement of their maturity. One child does better at the tests than another of the same age, it is said, only because of being more precocious" (1927, p. 90). Though this statement implies the comparison of children of the same age, Spearman's groups typically have not been children of the same age, and he has not resorted to a partial correlation technique to reduce his data to a constant age basis. Certainly race and nurture have not been partialed out, and only very occasionally has sex been experimentally treated as a separate factor. It is regrettable that this very fundamental matter[4]

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[4] It may be that neglect of nurture as a factor has been caused by a belief that its influence is specific. We find one of Spearman's students (Slocombe, 1926) stating: "How is it then, that practice, though present and producing an increase in score, does not influence g as measured? It is inferred that the influence of practice is specific to the form of test. Thus when a number of test forms are combined, practice enters as an uncorrelated specific factor."

The data of Slocombe, based upon a growth or nurture period of three months only, are most inadequate for so important a conclusion. The contradictory view is indicated upon many counts in the present writer's study, "The Influence of Nurture upon Native Differences" (1926), which includes data extending over a considerable range of years.

of maturity has not been thought worthy of prolonged examination. On *a priori* grounds why should one consider it of less significance in connection with intellect than with bodily structure?

The second factor which the writer finds clearly indicated in his own data, and which is undoubtedly present in the tests measuring *g*, is a verbal factor, for the tests that Spearman regularly uses as the better measures of *g* are very similar to those in which the writer has found a large verbal factor. One might say that this is merely quibbling over terms, and that what the present writer means by a verbal factor is what Spearman means by *g*. This does describe the situation in part, though the writer finds that the verbal factor is more limited in its scope than the statements of Spearman would indicate his *g* to be. It is nevertheless probably true that fully one-half of Spearman's *g* is represented by what is here called a verbal factor.

A very recent study by one of Spearman's students (Davey, 1926) gives data from which the author concludes that there is no general verbal factor. Dr. Davey reports the distribution of tetrads of the form $t_{1\,3\,2\,4}$, in which x_1 equals the score on an oral test, x_3, that on a second oral test, x_2, that on a pictorial test, and x_4, that on a second pictorial test. The median of such tetrads is .021, and the probable error (of the distribution, not of the median) by Spearman's Formula 2 is .019. This clearly shows a verbal factor, but Davey, picking out four of the eight verbal tests which have contributed the greatest to the creation of this median difference, finds that the median of the distribution of the remaining tetrads is approximately zero. This process of selection is unwarranted, for the very act of *a posteriori* selecting the four tests has capitalized chance in one direction, that of not yielding a verbal factor. We are surely warranted in disagreeing with Davey and in fact in citing his data as indicating a verbal factor of fairly wide extent.

Finally, there may be a third factor—not variability, in maturity, in sex, in race, in nurture; and not verbal—present in Spearman's *g*. The writer believes that if such a residual factor remains, after an allowance for the things mentioned, it is very small. A general factor, not verbal, is found by the writer throughout his work, but as he has not allowed for sex, race, or nurture, and probably not adequately for maturity, it is truly an open question whether any *g* factor at all would exist if these things had been properly taken into account.

The data which most adequately take into account maturity and sex is the Army Alpha data quoted very disparagingly by Spearman. As to the facts we can all agree that a single general factor is not indicated. As to the cause, Spearman writes that by the procedure followed, "the subjects, and still more so the testing, must have become heterogeneous to the last degree." Knowing something of the care with which this testing work was done, the writer does not believe the second of these charges is justified. As to the first charge, it is true that the group was rather heterogeneous, but this fact would introduce a *g* factor, not take out one already there. In brief, these data, almost unique in that they allow for maturity and sex, for the group consisted of adult men, do not yield a comprehensive *g* factor. A further study of the Army Alpha data is made in chapter ix below.

The relationship pictured by Spearman between education and *g* is stated in the following words: "On the whole, the most reasonable conclusion for the present appears to be that education has a dominant influence upon individual differences in respect of *s* (specific factors), but normally it has little if any in respect of *g*" (1927, *Abilities,* p. 392). The writer has shown in an earlier study (1927, *Interpretation*) that there is a great community of function between general intelligence and general scholastic achievement. If Spearman is correct in his statement, then general scholastic achievement is of necessity little affected by education. Though this may be so to an extent not ordinarily suspected, at least in certain respects it has been found by the writer not to be the case (Kelley, 1926, *Influence*). Further data and comment upon this important point are given below in chapter vi.

From Spearman's great dependence upon a central fund of intellective energy as a highly important category of mental life to a view wherein no general factor exists is indeed a far step, but one quite within the realm of possibility, judging by all the data at hand.

The nurture element earlier mentioned as a source of a general factor would be a general nurture, i.e., a tendency of the environment to stimulate or repress all intellectual development. Over and above this there may be a nurture operating on particular phases only of intellectual life, for example, an environment tending to stimulate or repress number ability. In either case there is no way, except by studying different age groups, to differentiate between native and acquired traits. Accordingly, with reference to the group traits found, we must, while noting Spearman's findings, be content to be concerned with the question of their existence rather than of the original or acquired nature of their origins.

A trait designated "perseveration" is considered by Spearman to be a universal factor. It is an expression of his fundamental "law of inertia." The sensory tests (speed of rotation of a color disk to cause fusion; seconds needed for adaptation to darkness) employed by Wiersma (1906) showed rather systematic differences in the case of 11 maniacs, 9 normals, and 18 melancholics. Heymans and Brugmans (1913) with these same tests and with others obtained, in the case of 15 normal subjects, such low correlations that no very likely conclusion is indicated from their data. Next, Wynn Jones (see Spearman, 1927) tested "77 children about 12 years of age" with tests, all of which involve motor activity and habits. The attempt was made to eliminate the motor factor by employing other motor tests not involving "perseveration," but this should surely be called unsuccessful, because the inter-correlations were very low, averaging .09, which we may expect to have been due largely to the unreliability of the tests. Spearman writes, "nothing of this diminutive size could possibly account for—or even by being eliminated sensibly diminish—the correlations shown in the foregoing table." This is true but altogether insufficient, as the author of the correction for attenuation should know, for one cannot partial out a motor factor by partialing out the scores on motor tests of very low reliability. Finally, there is the study of Lankes (1914) involving 47 students in the Islington Day Training College (ages and sex not indicated). His array of tests is quite extensive, but his inter-correlations are so low, running from —.05 to .51, and averaging .22, and his population so small, that probable errors are very large. The data are hardly serviceable in proving the existence of a perseveration factor. From the Wiersma, Heymans and Brugmans, Jones, and Lankes data, Spearman not only deduces a "perseveration" factor, but he defines many of its characteristics. These data are surely a feeble foundation for so imposing a superstructure. The writer believes that factors other than *g* are indicated by these data, but that it is hazardous to say more than this.

A trait called "oscillation" is presented as a third universal cognitive factor. Its experimental foundation is perhaps a trifle more adequate than in the case of "perseveration," for the largest population studied consisted of "about 80 children aged about 12 years." Here the inter-correlations ranged from .00 to .44, with an average of .21. Somewhat more adequate intelligence measures were available for this group.

One further general factor, this time connected with the field of conation, is considered to be present. This is Webb's (1915) "persistence of motives," and Garnett's (1919) "purpose" factor. The difficulty of establishing this is greatly increased by the fact that it is deduced from personal judgments and not from objective test scores. However, the population was large, "200 students with an average age of 21 years," and the evidence quite clear-cut that there is more than one factor present—the factors being *g* and "persistence of motives" according to Webb, and *g*, "purpose," and "cleverness" according to Garnett. A thing much to be hoped for is the measurement of these factors in a more objective manner, and a more exact establishment of their place in mental life.

Let us now note briefly other factors which Spearman calls group factors, because he considers them of less universality than the general factors. There has been no criterion established justifying, say "perseveration" as a general factor, and "memory" as a group factor. In either instance, and also in the case of *g*, the evidence of the generality of the factor depends upon the measures employed in the investigation. With reference to a designated set of measurements through all of which runs a certain factor *A*, through several of which runs a factor *B*, and in one of which is a factor *C*, the writer sees a value in the terms general, group, and specific; but this value is only in the descriptive power of these terms for the particular situation which is being investigated. Given other tests, the *A* factor might become specific, the *B* factor become general, and the *C* factor become a group factor. To determine factors which are not thus dependent upon a particular set of tests, it is necessary to utilize many measures at once, thus making a very exhaustive survey of the mental ability of the subjects tested. In noting factors reported by Spearman additional to those already mentioned we will not draw a distinction between general and group factors.

There is a memory factor certainly extending to different sensory fields and to verbal material.

One of Spearman's "ideal" relations is "conjunction," and he finds a factor of this nature. The writer prefers to call it a "numbers" or "arithmetical" factor as being more descriptive of the content and nature of the tests revealing it.

There is a constructive mechanical ability factor which may be related to that characterized by the writer as "manipulation of spatial relationships."

Dealing with this trait, McFarlane (1925) reports that the factor is found in the case of boys and not in that of girls. Certain other evidence of sex differences is cited, but no extensive treatment of sex as a factor in mental life is undertaken by Spearman. There is a great deal of literature bearing upon this matter. Noteworthy in this is the study already made by Terman (1925), as well as a further investigation by him now under way. They suggest rather far-reaching mental differences which are correlated with sex.

The reader must not conclude because of the criticisms that have been made of Spearman's technique and interpretation that there is wide disagreement between his findings and those of the present writer. On the whole the two sets of findings are quite remarkably in harmony, the agreements being in the matter of a spatial, a numerical, a memory, and even a general factor, though this last is differently interpreted, and also in the conclusion that a large number of specific motor (probably also sensory) factors exist. There is scarcely a disagreement in the matters of music, purpose, cleverness, and sex, though here the data are inadequate. There does seem to be a real disagreement in the importance and extent of a verbal factor and in that of a mental speed factor.

In addition to the contributions to this problem of mental life made by Spearman and his students, and by such continental workers as are referred to by him, attention should be called to the work of a considerable number of other students, particularly in America (for citation of these, see Kelley, 1927, *Interpretation*, chap. iv, dealing with "The Determination of Idiosyncracies"), and also a rich literature, though statistically rather difficult to handle, bearing upon racial differences. Certain of these studies not earlier reviewed by the writer (1927, *Interpretation*) are scrutinized in some detail in chapter ix below. Preceding the discussion of these, the main body of new data collected by the writer is given in chapters iii to viii following.

It is admitted that the treatment of the present chapter and related treatments by the writer (Kelley, 1927, *Interpretation*, chap. iv) are an inadequate discussion of psychological points of view, varied both as to phenomena dealt with and methods employed. They may, however, suffice to emphasize the variety of mental activity of which man is master and to outline a picture of mental life which future study will fill in. In brief, the boundaries of mental traits are ruts, not far-flung indefinite fringes of consciousness. Mental life does not operate in a plain but in a network of canals. Though each canal may have indefinite limits in length and depth, it does not in width; though each mental trait may grow and become more and more subtle, it does not lose its character and discreteness from other traits.

CHAPTER II

TRAITS AS GROUP PHENOMENA

One of the most common findings of mental measurement has been the fact that, generally speaking, correlation exists between the scores of different mental tests. Further, if "good," "high," or "excellent" scores receive higher numerical values than "poor," "low," and "unsatisfactory" scores, then it has generally been found that the correlation between scores is positive. This is so common a finding that the writer will not stop to cite evidence. It has been found for groups which were as heterogenous as one composed of tramps and university graduate students, as well as for groups as homogeneous as those of a single age, or those found in a single school grade, though in this latter case the correlations found between different mental tests have been fairly low. A number of causes may contribute to these universal findings, such causes, for example, as the use of groups of heterogeneous (*a*) maturity, (*b*) racial origin, (*c*) nurture, (*d*) sex, also (*e*) the poor selection of mental tests employed, or (*f*) an intrinsic similarity in all mental activities. In so far as (*a*), (*b*), (*c*), (*d*), and (*e*) are the causes, (*f*) is not. In the sense in which here used, a number of tests which were very similar to each other would be called a "poor" selection. The pertinence of the first four causes may be pointed out by assuming that there exist at least two independent mental traits and then noting that heterogeneity in populations studied, introduced through (*a*), (*b*), (*c*), or (*d*), will presumably lead to positive measures of correlation between the two intrinsically independent traits. The accompanying numerical example in Table I illustrates case (*a*).

If each child is measured at that age at which his maturity is one-half of his adult maturity, he will receive the scores in the two traits as given in columns 2 and 3, and as may be easily shown by calculation the correlation between these two traits is zero. Now let us suppose that maturity in the first trait goes with maturity in the second; for example, if at a certain time a child possesses

x-proportion of his adult arithmetic reasoning ability he likewise possesses y-proportion of his adult reading ability, and when a second child possesses x-proportion of his adult arithmetic reasoning ability he likewise possesses y-proportion of his adult reading ability, etc. To make the numerical illustration simple we will here

TABLE I

Hypothetical Scores of Ten Children on Two Intrinsically Independent Traits

(1) Individuals	(2) Trait 1: Score at a Maturity Level Equal to ½ That of Adult	(3) Trait 2: Score at a Maturity Level Equal to ½ That of Adult	(4) Maturity Influence at Time of Measurement	(5) Resulting Trait 1 Score	(6) Resulting Trait 2 Score
A........	8	9	—1	7	8
B........	9	10	0	9	10
C........	9	11	1	10	12
D........	10	8	2	12	10
E........	10	12	—2	8	10
F........	10	10	—1	9	9
G........	10	10	0	10	10
H........	11	10	1	12	11
I........	11	11	0	11	11
J........	12	9	0	12	9

assume perfect correlation between the proportions x and y as age changes, but to establish the point at issue it is only necessary to suppose that there is positive correlation between them. Column 4 contains the entry (—1) for the maturity measure pertaining to individual A. This states that when measured he was less mature than the average of the group of 10. Adding this (—1) to his Trait 1 and again to his Trait 2 scores, we obtain the values 7 and 8 as given in columns 5 and 6. Thus if individual A is young as judged by the average maturity of the group when he is tested, he makes a score of 7 on the first test and of 8 on the second, not the scores of 8 and 9 which he would make if tested a little later in life. In a similar manner we get scores 9 and 10 in columns 5 and 6 as the obtained scores for individual B; 10 and 12 as the obtained scores for individual C; etc. Calculation of our hypothetical data shows that columns 5 and 6 are quite substantially correlated. We accordingly see from it that individuals possessed of two traits between which there is zero correlation, if tested at the same stages of maturity, show traits which are correlated when the individuals of the group are tested at different levels of maturity, provided only that there is correlation between the maturity of the two traits. Now the writer is unaware of any mental traits in which growth during the days of childhood does not take place, and thus for all traits in which the status of the child is low in very early life and high at adulthood there is a necessary correlation between the maturity of the traits.

The reader can readily satisfy himself that race heterogeneity, providing the individuals of the one race are on the average lower as adults in both the two traits considered than those of the other race, operates to introduce correlation just as do unequal levels of maturity. Similar observations apply to sex and nurture. We may therefore conclude that even if the selection of the two mental traits is excellent in the sense that the mental capacities are intrinsically independent, positive correlation will nevertheless commonly be found between them, owing to the heterogeneity of population employed.

Though it is obvious that the correlation due to differences in maturity, in racial stock, in sex, and in nurture should be ruled out, it is by no means clear just how far one should go in securing homogeneous experimental groups. It is to be expected that 10-year-old boys are not all of the same degree of maturity, that is, they have not accomplished the same fraction of their adult standing. This, however, seems to be a relatively unimportant point, for children so far asunder as imbeciles and children of IQ 150 have growth curves in which the fractions of adult mental stature attained at successive stages are approximately the same. Accordingly, one may substantially eliminate the maturity influence by choosing age-homogeneous groups.

The securing of groups which are homogeneous from the racial standpoint offers greater difficulty. In the study herewith reported American white school children in a single school grade have been thrown together to provide a single group. In a more extended further study this should not be the case, for the sexes should be treated separately, those of different ages in the same grade separately, and those of the same age in different classes brought together, and still other steps taken to secure homogeneity. The sexes were thrown into a single group in the present study to secure larger populations, but the writer is led to believe, particularly because of certain as yet unpublished findings of Dr. L. M. Terman, that either the sexes should have been treated separately, or an independent variable, sex, should have been introduced and later partialed out.

Would children all of the same race, same age, and same sex constitute a homogeneous group? Would children of the same age, same race, same sex, and same general level of parental intelligence constitute an appropriate group? Would children of the same race, same age, same sex, and same parents provide the group desired? Presumably this latter would be too homogeneous, because any dominant trait through either parent would yield similar measures in offspring, whereas the trait might be very independent as judged by other members of the same race. To illustrate, let us suppose that a certain type of musical ability is dominant. The children of parents one of whom is duplex in this trait all possess this ability. If it shows equally in each child and if the family of sibs constitute the experimental group, there would be no evidence that any of them were superior in this trait. Of course no one is superior to the rest, but all may be greatly superior to the rest of mankind; and thus the very fact we wish to know is concealed, owing to the extreme homogeneity of the group.

Let us suppose a second situation: Two races living together, the one being superior in Trait *a*, let us say music, and the other in Trait *b*, let us say mathematics. If there is great difference between the two races in these two traits, the fact that they live in the same community does not mean that they compete with each other in the matter of musical and mathematical activities. In general, the members of the first race will forswear mathematics and compete among themselves in music, and the second race will permit the first race to provide the musical entertainment while they enjoy the mathematical offerings of society. In other words, living in the same community is not sufficient warrant for throwing two peoples together into a single experimental group. Rather, if the members of the two races are to be placed in a single group when studying Traits *a* and *b,* it should be because the mean adult accomplishments of the two races are substantially the same in these two traits.

Throwing together races as described, one's findings would of course be with reference to such groups; and this is as it should be. We cannot establish facts of heredity, unitary traits, etc., as true per se quite independent of the culture from which the data have come. If all the inhabitants of Mars, granting there are such, have the musical ability of Wagner they will not discover that they are exceptional in musical ability; and if a guidance expert from this earth were to drop in upon them and give the Seashore Musical Ability and the Terman General Intelligence tests he would make a rather sorry job of counseling if he advised everyone to follow music as a vocation, for probably even Martians could not live by music alone. The Volga boatmen may all be more musical than the bandmasters of North America, but if they must live along the Volga they should be boatmen and not bandmasters. These observations are made to justify the use of a group mean as a point of reference in the present study and in any study having as its purpose the discovery of unitary mental traits.

As pointed out, a fraternity is too homogeneous a group; and the entire population of mankind is too heterogeneous because this latter includes groups which are not in fact competing cultures. Travelers in the Near East report that the Turks and Armenians, though living in the same territory, engage in different pursuits and that each establishes his own social setting. One making an examination of these two races for a number of mental traits would undoubtedly find differences in racial mean scores. Such a finding would first give us evidence that the two races should not be combined in a single race study; and, provided the one race did not hold a higher general level in all traits than the other, it would, secondly, give us evidence as to what were independent mental traits. A study of racial differences can be made the means of determining differentiable mental abilities, as can the study of idiosyncracies within a homogeneous racial group.

It would seem that the findings of the two types of study would of necessity point in the same direction, for if two traits, say music and mathematics, are independent in the members of a race *A,* it is rather difficult to think of them as being other than biologically independent in race *B*. Also, if independent in race *A,* it would be possible to breed by selection from race *A* a new race *A′*, which would have different mean scores in these two traits than the mean scores of race *A*. Having done this, race *A′* and race *A* will show racial differences which are manifest in connection with the same traits as were the unitary traits in race *A*. Such a result as this can, of course, only happen with reference to traits which are, in part at least, independent of each other. A study of racial differences would reveal traits which, in part at least, are independent, while the study of idiosyncracies within a homogeneous race can reveal traits which are completely independent. These two types of study should never be in conflict, though the latter can go farther in the matter of mental analysis than the former. Though they throw light upon the same issue one should endeavor to keep the two types of study distinct. The individual study can be freed of a racial bias if the races combined into a single group are such as have at a given age the same mean levels of attainment in the traits in question. Here in America we can throw together a number of racial stocks and approximate this condition.

The reader is not to think of this as an empirical sort of procedure, removing the study from recognized biological practice. This approach does not, in fact, constitute a difference from that followed in the very valuable heredity studies which have been conducted in the animal and plant kingdoms. The thing that constitutes a trait in the study of the vinegar fly is something which has been found to vary from the "wild type" condition. The wild type is a point of departure in the studies of drosophila as is the existing cultural level in studies of human character traits. This argument forces us to the conclusion that what constitutes a trait of a human being depends upon the other human beings with whom he comes into contact. A certain Jew would have certain traits in Central Africa, others in the Ghetto in New York, and still others in Peking. This may seem confusing, since the person himself has not changed.

There are two alternatives, neither of which involves this change in trait with change in location. One point of view would be to use the mean of all the races in the world as the point of departure, so that when a person is once defined in terms of this world-wide standard no re-definition is necessary attendant upon a change in geographical location. This procedure has the very serious drawback that when interpreting the status of an individual of a certain racial or national culture two things must be known: (*a*) the relation of the individual to the world-wide standard; and (*b*) the relation of the culture to the world-wide standard. It thus seems that the world-wide standard is one derived from too wide a cultural group to be meaningful for national and local problems.

The other alternative is to take one's self as a standard and compare each and every other person with one's self. Then, so far as one's own interpretations of a second person are concerned, no change is demanded in passing over national or cultural boundaries. This method, though perhaps very common, is not serviceable as a method which can be made objective, passed from individual to individual, and used as a basis of general social understanding.

Finally it may be noted that the groups used in this study—American white children in a given school grade—do not constitute poor samplings of what would correspond to educational cultural groups in America, though sex, age, and race factors are not allowed for in the best manner possible. Though these factors are not large they should be more adequately taken into account in a further study. From the standpoint of strict biological treatment it is unfortunate that we do not have a homogeneous, stable, original human type, such as is the wild type in connection with drosophila, to serve as a point of departure for all trait investigations. In the case of the vinegar fly the outstanding characteristic of the wild type is its stability. It is questionable whether anything which might be called a wild type in the human species possesses this characteristic. One very stable feature of the human civilization of this country, possibly the most stable, is that represented by the average of the white population, so that we can scarcely do better than intentionally choose a white group to work with, one homogeneous with respect to age and sex and random with respect to race, except that all races known to have different mean attainments should be kept separate.

It is to be noted that the negro population is not included in a group as described, since for many traits, especially those represented by our common general intelligence and psychological tests, it is known that substantial race differences exist between American white and negro samplings. The stability of the average based upon a sampling of both populations would be less than that based upon but a single population; and, further, since, in the main, American white adults compete with American white adults, this will constitute a more homogeneous cultural group than a population of negroes and whites. The same argument applies though with less force to an admixture of certain other races in the United States, especially to any which are largely concentrated in particularly limited geographical locations. Southwest Texas, having many Mexicans; certain valleys in California, having many Italians; Jewish districts in New York; German districts in Missouri, are not all fair samples of American white population. This matter is probably not of great importance if the races thrown together all have the same mean adult measure in the traits considered, but it becomes of prime importance where this is not the case. It would be desirable in a future study of traits to study the racial origins of each and every individual in the population dealt with—then, if a critic postulates race as the cause of any of the findings, he has at hand the means of testing the matter.

Having selected a group to work with the problem is to find as many as possible uncorrelated mental traits. The uses to which such information could be put are innumerable. Not only are such data sufficient for the understanding of human character, but they are absolutely necessary. Let us suppose that musical ability and mathematical ability are uncorrelated and suppose we have a test measuring each in equal proportion and yielding a single score. Thus if a child scores above the average in this test we do not know whether it is due to a superiority of musical ability or of mathematical ability, or superiority in both. Separate scores on the two traits are absolutely necessary if a correct understanding of the child is to result. There is no short road yielding equally valuable information at less labor, and any trait not discovered and not separately measured will always be a scource of uncertainty and confusion. After we know that musical ability and mathematical ability are independent of each other and secure reliable measures of each but (supposing it is) do not know that memory ability is independent of each of the other two and secure no measure of it, we will with reference to every interest, study, vocation, or profession, making special demands upon memory ability, be groping in the dark. In other words there is no substitute for knowledge concerning this specific trait, because this trait is independent of the other traits measured.

Again, if we have knowledge as to traits which are correlated but not perfectly with each other the possibility of correct interpretation is present, but the difficulty of making such correct judgments is greatly augmented. Suppose the three abilities already mentioned are independent mental traits and that we have three measures, *A, B,* and *C,* such that *A* is 50 per cent musical ability, 25 per cent mathematical ability, and 25 per cent memory ability; that *B* is 25 per cent musical ability, 50 per cent mathematical ability, and 25 per cent memory ability; and that *C* is 25 per cent musical ability, 25 per cent mathematical ability, and 50 per cent memory ability. The correct understanding of the individual is no longer to be based upon scores in tests as before (where we were able on the basis of the musical ability test alone to define the status of the individual in music with reference to the group and similarly for the other two traits), but upon differences between scores in tests. Furthermore the differences are weighted differences and the determination of the weightings is none too easy a task, nor is their use after determination. The advantages of measures of traits which are independent of the other traits involved are so great for all problems of guidance, classification, and education that they are, in truth, at the foundation of a new psychology which the future is to build.

It ordinarily happens that tests as drawn up are of the nature of the tests *A, B,* and *C,* each measuring more than a single mental function. The problem is then twofold: first, a determination, having tests *A, B,* and *C,* of what the independent mental traits are; and secondly, an experimental construction of new tests measuring these independent traits. This latter has not been accomplished in any real or completely satisfactory manner in the experimental investigation here described, and remains a larger problem to be solved in the future. The present study does, however, make considerable progress in throwing light upon the first phase, and thus it paves the way for the next step. We may now turn to the mathematical treatment which, given such tests as *A, B,* and *C,* enables us to ascertain whether one or more underlying traits are present.

CHAPTER III

THEORY AND STATISTICAL TECHNIQUE

This chapter is devoted to proving certain basic propositions and to deriving several much-needed formulas for probable errors.

PROPOSITION 1.—*A number of traits greater than 1 may be involved in the correlation between two mental measures, no matter what the value of the correlation coefficient.*

To prove this it is only necessary to show that it is possible to conceive of two independent variables underlying the two measures which are correlated. Let x_1 equal the score of an individual in Trait 1 as a deviation from the group mean, and let σ_1 be the standard deviation of such scores. Let x_2 equal the score of an individual in Trait 2 as a deviation from the group mean, and let σ_2 be the standard deviation of such scores. When further variables are involved x_3, σ_3, x_4, σ_4, etc., are defined in a similar manner. Let r_{12} equal the product-moment correlation between x_1 and x_2. Let us divide x_2 into two parts thus:

$$x_2 = c_1x_1 + c_1e_2$$

a part which is perfectly correlated with x_1, namely, c_1x_1; and a part which has zero correlation with x_1, namely, c_1e_2. Let us under these conditions ascertain the numerical limits in the values of r_{12}. The quantities x_1 and e_1 are variables, and c_1 is a numerical constant lying between the limits $\pm\,\sigma_2/\sigma_1$, for the extreme values that c_1 can take exist when $\sigma_{e_2} = 0$. In this case

$$c_1x_1 = x_2$$

$$c^2{}_1\sigma^2{}_1 = \sigma^2{}_2$$

$$c_1 = \pm\frac{\sigma_2}{\sigma_1}$$

For the correlation between x_1 and x_2 we have

$$r_{12} = \frac{\Sigma\, x_1(c_1x_1 + c_1e_2)}{N\sigma_1\sigma_2} = \frac{\sigma^2{}_1c_1}{\sigma_1\sigma_2} = \frac{\sigma_1c_1}{\sigma_2}$$

The largest value for r_{12} is found when $c_1 = \sigma_2/\sigma_1$. In this case $r_{12} = 1$. The smallest value is found when $c_1 = -\sigma_2/\sigma_1$ and then $r_{12} = -1$. Accordingly, whatever the value of r_{12} we may think, if we so desire, of the correlation as due to not over two independent traits. The two independent traits which we chose in the illustration were x_1 and e_2. We might have interchanged the variables, choosing x_2 and e_1. Of these two independent traits one is one of the two measured traits. Thus it is always possible, if no further information than the coefficient of correlation between two variables is known, to look upon the correlation as due to two independent traits. It frequently is not desirable so to do. This is especially the case when it is known that a part of each of the two variables is chance. Suppose a situation in which the variables may be written

$$x_1 = x_\infty + e_1$$

$$x_2 = x_\omega + e_2$$

in which x_1 is the obtained score in the first trait, x_2 in the second trait; x_∞ the true score in the first, x_ω in the second; and e_1 the chance factor entering into the measure of the first trait, and e_2 the chance factor entering into the measure of the second trait. This situation parallels many actual experimental situations. It is no longer desirable to think of the correlation between x_1 and x_2 as due to two independent measures, for one of these measures would involve as a part of itself a purely chance element. It may, however, be desirable to think of the correlation between x_∞ and x_ω as due to two independent measures, let us say x_∞ and that part of x_ω that is independent of x_∞. We may call it x_τ. Then

$$x_1 = x_\infty + e_1$$

$$x_2 = c_2 x_\infty + x_\tau + e_2$$

in which x_∞, e_1, x_τ, and e_2 are independent variables. Obviously following from the preceding proof it is always possible to interpret the correlation between two variables in this manner; that is to say, the correlation may be thought of as due to four independent variables, two of which are chance, and one of which is that part of one of the measures which is not chance and which is not correlated with the true part of the other measure. Statistically there is no means of differentiating between x_T and e_2 unless some other measure, say x_3, enters into the problem, for e_2 correlates with neither x_∞ nor e_1, and x_T likewise correlates with neither of these.

If we look to further variables than x_1, x_2 as a means of differentiating between x_∞ and e_1, or between x_ω and e_2, or between x_T and e_2, we can readily find the necessary two added variables to enable us to do so. Let us call x_I a measure of the same thing that x_1 measures, and let us say that the method of derivation of x_I assures us that it is a measure equal in excellence to x_1. Then we may write

$$x_I = x_\infty + e_I$$

in which x_∞ is the identical measure entering into the right-hand member of the equation giving x_1, and in which

$$\sigma_{e_I} = \sigma_{e_1}$$

thus giving $$\sigma_I = \sigma_1$$

and, further, for the correlation between x_1 and x_I we have

$$r_{1I} = \frac{\Sigma(x_\infty + e_1)\ (x_\infty + e_I)}{N\sigma_1\sigma_I} = \frac{\sigma^2_\infty}{\sigma^2_1} \ldots\ldots\ldots\ldots [3]$$

Since $\sigma^2_1 = \sigma^2_\infty + \sigma^2_{e_1}$, we immediately obtain

$$\frac{\sigma^2_{e_1}}{\sigma^2_1} = 1 - r_{1I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [4]$$

Thus, though we have not separated for each individual the x_∞ and e_1 factors, we have for the group fully determined how much of the variance is representative of the real underlying trait and how much of it is due to chance. If the total variance is σ^2_1 the proportion due to chance is $(1 - r_{1I})$ and the portion due to a real trait is r_{1I}.

If x_{II} is a second measure similar to x_2 we may determine σ_{e_2} in a manner similar to that which gave us σ_{e_1}. Thus, knowing $\sigma^2_\infty/\sigma^2_1$ and $\sigma^2_{e_2}/\sigma^2_2$ we immediately find σ^2_T/σ^2_2. The steps are

$$r_{12} = \frac{\Sigma(x_{\infty} + e_1)\,(c_2 x_{\infty} + x_T + e_2)}{N\sigma_1\sigma_2}$$

$$= \frac{(\sigma_{\infty})}{(\sigma_1)}\,\frac{(c_2\sigma_{\infty})}{(\sigma_2)}$$

$$r_{1I} = \left(\frac{\sigma_{\infty}}{\sigma_1}\right)^2$$

$$r_{2II} = \frac{c^2{}_2\sigma^2{}_{\infty}}{\sigma^2{}_2} + \frac{\sigma^2{}_T}{\sigma^2{}_2}$$

$$\frac{\sigma^2{}_T}{\sigma^2{}_2} = r_{2II} - \frac{r^2{}_{12}}{r_{1I}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [5]$$

In this text, when the expression "the proportion of a variable due to chance" is used, it means the ratio $\sigma^2{}_{e_1}/\sigma^2{}_1$. More accurately stated this is the proportion of the variance of the variables that is to be attributed to chance, for this proportion will presumably not hold for any particular individual whose score might be under consideration.

An interesting situation which not uncommonly arises exists when $\sigma^2{}_T$ equals zero. Professor Spearman found this to be the case so commonly that it led to his general-factor theory, which is that any intellectual activity may be thought of as due to a single factor underlying this and all other intellectual activities plus a factor specific to the trait in question and *not found in any other but closely allied traits.* If Spearman had omitted the idea represented by italics, leaving the interpretation in fact erroneously attached by the present writer and certain others to Spearman's early wording of the theory, the proof of the inadequacy of the theory to fit many facts of inter-correlation of mental tests would have been in general quite simple. Including the part italicized the theory is quite impregnable because just as soon as certain tests do not give correlations which can be adequately explained by a single factor plus specific factors, then it may be said that some of the tests are measures of "closely allied traits." The writer therefore does not set himself the task of proving or disproving Spearman's general-factor theory (also called Spearman's theory of two factors—a general factor and a specific factor), but rather the quantitative problem of determining what mental traits are such that a single factor is sufficient to explain the correlations holding throughout, and what mental traits demand for their adequate explanation two or more general factors. We will first investigate the correlation conditions which must hold if a single general factor is sufficient to account for the correlation between two variables.

PROPOSITION 2.—*One general trait plus factors specific to each of two variables is always sufficient to account for the correlation between two variables, no matter what its numerical value.*

Let the gross score in the first measure be X_1. Let this be a function of a trait A also present in X_2, and of a trait E_1 not found in X_2. Then we may write

$$X_1 = f\ (A_1 E_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [6]$$

Now if M_1 is the mean of the X's for the population in question; M_a the mean of the A's; M_{e_1} the mean of the E's, let us write

$$x_1 = X_1 - M_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [7]$$

$$a = A - M_a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [8]$$

$$e = (E_1 - M_{e_1}) \text{ times some constant} \ldots [9]$$

If X_1 is some trait, such for example as computation ability, which grows in the single individual from time to time and changes from individual to individual in an essentially homogeneous group in a manner not markedly saltatory, and if M_1 does not approximate zero, i.e., is not small with reference to σ_1, then Taylor's Series may be employed, yielding

$$x_1 = c_1\, a + e_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [10]$$

in which c_1 is some constant not changing as we pass from member to member of the group in question. Likewise

$$x_2 = c_2\, a + e_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots [\textit{See}\ 10]$$

The specific factors are e_1 and e_2. The common factor is a. The independent variables a, e_1, and e_2 are uncorrelated, and c_1 and c_2 are constants. We may set $\sigma_a = 1$ without any loss in generality because c_1 and c_2 are still at our option. It is easily shown that

$$r_{12} = \frac{c_1 c_2 \sigma^2_a}{\sigma_1 \sigma_2} = \alpha_1 \alpha_2 \qquad [11]$$

where the α's are defined by the equations

$$\left.\begin{aligned} \alpha_1 &= \frac{c_1 \sigma_a}{\sigma_1} \\ \alpha_2 &= \frac{c_2 \sigma_a}{\sigma_2} \end{aligned}\right\} \qquad [12]$$

and it is immediately obvious that the limits of either α are ± 1, so that the limits of α^2_1 or α^2_2 are zero and 1. The quantity α^2_1 will be referred to as a variance—it is the proportion of the total variance, σ^2_1, due to the factor a. We may write

$$r^2_{12} = \alpha^2_1 \, \alpha^2_2$$

Substituting the lower and upper limits we find that r^2_{12} must lie between zero and 1, i.e., it may take any value. Our conclusion therefore is that no matter what the value of r_{12} it is conceivable that it was consequent to two variables having one general factor and specific factors. This statement substantiates Proposition 2, and it further shows that it would be entirely futile to attempt to test Spearman's general-factor theory by means of a two-variable problem. We must concern ourselves with at least three variables.

PROPOSITION 3.—*Three variables may be thought of as due to a single general factor plus one specific factor only in case two of the variables are perfectly correlated.*[1]

[1] The statement of this and subsequent propositions would be more accurate if preceded by the words "judging from the inter-correlations only." Since in this study all conclusions reached are based upon product-moment correlation coefficients, this phrase is omitted for the sake of conciseness.

Let

$$x_1 = c_1 a$$

$$x_2 = c_2 a$$

$$x_3 = c_3 a + e_3\,;$$

then $\alpha^2_1 = 1$; $\alpha^2_2 = 1$; and $r^2_{12} = \alpha^2_1\,\alpha^2_2 = 1$. Thus proving the proposition.

Proposition 4.—*Three variables may be thought of as due to a single general factor plus two specific factors only in case the product of two of the three inter-correlations is equal to the third.*

Let

$$x_1 = c_1 a$$

$$x_2 = c_2 a + e_2$$

$$x_3 = c_3 a + e_3\,;$$

then $\alpha^2_1 = 1$; $r_{12} = \alpha_1\alpha_2$; $r_{13} = \alpha_1\alpha_3$; $r_{23} = \alpha_2\alpha_3$. Therefore,

$$r_{12}\,r_{13} = r_{23} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [13]$$

or

$$r_{12}\,r_{13}/r_{23} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots [14]$$

This establishes the theorem. That this criterion may be serviceable we require the probable error of the function $r_{12}\,r_{13}/r_{23}$. It is derived herewith.

Let

$$f = \frac{r_{12}\,r_{13}}{r_{23}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots [15]$$

Taking logarithmic differentials we have

$$\frac{df}{f} = \frac{dr_{12}}{r_{12}} + \frac{dr_{13}}{r_{13}} - \frac{dr_{23}}{r_{23}}$$

Squaring and summing and dividing by the population gives

$$\frac{\sigma^2_f}{f^2} = \frac{\sigma^2_{r_{12}}}{r^2_{12}} + \frac{\sigma^2_{r_{13}}}{r^2_{13}} + \frac{\sigma^2_{r_{23}}}{r^2_{23}} + \frac{2\sigma_{r_{12}}\,\sigma_{r_{13}}}{r_{12}\,r_{13}}\, r_{r_{12}\,r_{13}}$$

$$- \frac{2\sigma_{r_{12}}\,\sigma_{r_{23}}}{r_{12}\,r_{23}}\, r_{r_{12}\,r_{23}} - \frac{2\sigma_{r_{13}}\,\sigma_{r_{23}}}{r_{13}\,r_{23}}\, r_{r_{13}\,r_{23}}$$

The formula for the correlation between correlation coefficients (see Formula 128 of Kelley, *Statistical Method*) may be utilized to reduce this to an expression involving r_{12}, r_{13}, r_{23}, and N, the population. The resulting expression does not greatly simplify, and as the values of $(N\sigma_{r_{12}}\,\sigma_{r_{13}}\,r_{r_{12}\,r_{13}})$ are fairly readily obtained by Formula 66, it is easier to reduce the standard error to the following form and use Formula 66.

$$\sigma_f = \frac{f}{\sqrt{N}}\left\{\left(\frac{1}{r_{12}} - r_{12}\right)^2 + \left(\frac{1}{r_{13}} - r_{13}\right)^2 + \left(\frac{1}{r_{23}} - r_{23}\right)^2\right.$$

$$+ 2\,\frac{(1-r^2_{12})\,(1-r^2_{13})\,r_{r_{12}\,r_{13}}}{r_{12}\,r_{13}} - \frac{(1-r^2_{13})\,(1-r^2_{23})\,r_{r_{13}\,r_{23}}}{r_{13}\,r_{23}}$$

$$\left. - \frac{(1-r^2_{12})\,(1-r^2_{23})\,r_{r_{12}\,r_{23}}}{r_{12}\,r_{23}}\right\}^{\frac{1}{2}} \qquad [16]$$

PROPOSITION 5.—*Three variables may be thought of as arising from one general factor plus three specific factors when the absolute value of the product of every two of the three inter-correlations is less than the third and when one or three of the inter-correlations are positive.*

Let

$$\left.\begin{aligned} x_1 &= c_1a + e_1 \\ x_2 &= c_2a + e_2 \\ x_3 &= c_3a + e_3 \end{aligned}\right\} \qquad [\textit{See } 10]$$

in which as before a, e_1, e_2, and e_3 are uncorrelated. We will let

$$\left.\begin{aligned} \frac{c_1\sigma_a}{\sigma_1} &= \alpha_1 \\ \frac{c_2\sigma_a}{\sigma_2} &= \alpha_2 \\ \frac{c_3\sigma_a}{\sigma_3} &= \alpha_3 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots [\textit{See } 12]$$

It is readily found that

$$\left.\begin{aligned} r_{12} &= \alpha_1\alpha_2 \\ r_{13} &= \alpha_1\alpha_3 \\ r_{23} &= \alpha_2\alpha_3 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots [\textit{See } 11]$$

We may solve these three equations for α_1, α_2, α_3 as follows:

$$\left.\begin{aligned} \alpha^2{}_1 &= \frac{r_{12}\, r_{13}}{r_{23}} \\ \alpha^2{}_2 &= \frac{r_{12}\, r_{23}}{r_{13}} \\ \alpha^2{}_3 &= \frac{r_{13}\, r_{23}}{r_{12}} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots [17]$$

Since the part of the first variable which is specific to the first variable, namely, e_1, is not included in c_1a it is obvious that $c_1\sigma_a$ is always, in absolute value, less than σ_1. Thus $\alpha^2{}_1$, as well as $\alpha^2{}_2$ and $\alpha^2{}_3$, lies between the limits zero and 1. Whenever the last three equations yield positive values less than 1 for $\alpha^2{}_1$, $\alpha^2{}_2$, and $\alpha^2{}_3$ then the three variables x_1, x_2, and x_3 may be thought of as composed of one general factor plus specific factors. The variances $\alpha^2{}_1$, $\alpha^2{}_2$, and $\alpha^2{}_3$ will lie between zero and 1 when the quotients are positive and the numerators less than the denominators. This establishes

Proposition 5 as stated. These are very easy conditions to meet. In other words there is a wide range within which, judged by the correlations yielded, three variables may be conceived of as consequent to one general factor plus specific factors. The main value of Proposition 5 is as a means of quickly finding three variables which differ widely one from another. In order to be certain of a judgment in cases where α^2_1, α^2_2, and α^2_3 differ slightly from zero we should use the probable error of this quotient as already derived in Formula 16.

As Proposition 5 is only occasionally useful we need a requirement which is more difficult to fulfil. When may three variables be thought of as consequent to two independent general factors plus no specific factors?

Proposition 6.—*Three variables may be thought of as due to two independent general factors and no specific factors when the multiple correlation coefficients* $r_{1\cdot 23}$, $r_{2\cdot 13}$, $r_{3\cdot 12}$ *are each equal to 1.*

It may readily be shown that these conditions are met when

$$1 + 2r_{12}\,r_{13}\,r_{23} - r^2_{12} - r^2_{13} - r^2_{23} = 0 \quad \ldots\ldots\ldots [18]$$

Equation 18 is accordingly the required condition. We will prove it herewith: Let

$$x_1 = c_1a + k_1b$$

$$x_2 = c_2a + k_2b$$

$$x_3 = c_3a + k_3b$$

Eliminating a and b from these three equations will give us a linear equation in x_1, x_2, and x_3. Thus any one of these three variables may be determined from a knowledge of the other two. This is equivalent to the statement $r_{1\cdot 23} = \pm 1$; $r_{2\cdot 13} = \pm 1$; $r_{3\cdot 12} = \pm 1$.

The condition as stated is practically never met by raw correlation coefficients and the reason is undoubtedly due to the presence of specific or chance factors in variables x_1, x_2, and x_3. As applying to raw correlation coefficients this is not a valuable criterion and its probable error has not been calculated. If we apply this test not to raw coefficients of correlation but to those corrected for attenuation we eliminate as far as possible the chance factor. If x_∞, x_ω, and x_τ are the true scores of individuals whose obtained scores are x_1, x_2, and x_3, then x_∞, x_ω, and x_τ may be thought of as consequent to two general factors only when

$$1 + 2r_{\infty\omega}\, r_{\infty\tau}\, r_{\omega\tau} - r^2_{\infty\omega} - r^2_{\infty\tau} - r^2_{\omega\tau} = 0. \ldots\ldots\ldots [19]$$

The probable error of this expression has not been calculated and this test therefore has not been used in the present study. It may, however, prove a valuable aid as soon as its probable error is known.

Proposition 7.—*Three variables may always be thought of as due to two general factors plus one specific factor.*

Let

$$x_1 = c_1 a + k_1 b + e_1$$

$$x_2 = c_2 a + k_2 b$$

$$x_3 = c_3 a + k_3 b$$

Let us call $(x_1 - e_1)$ a new variable and designate it by x_4. Then $x_4 = c_1 a + k_1 b$. Dealing with x_2, x_3, and x_4 the condition laid down in the preceding proposition must hold if the three variables x_2, x_3, and x_4 are to be thought of as due to two general factors. We thus have

$$1 + 2r_{24}\, r_{34}\, r_{23} - r^2_{24} - r^2_{34} - r^2_{23} = 0$$

We must obtain a relation between r_{24} and r_{34} and the known values r_{12} and r_{13}. In addition to the α's as already defined let us here define β_1 and β_2 as follows:

$$\left.\begin{aligned} \beta_1 &= \frac{k_1 \sigma_b}{\sigma_1} \\ \beta_2 &= \frac{k_2 \sigma_b}{\sigma_2} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots [20]$$

It is readily shown that $r_{12} = \alpha_1\alpha_2 + \beta_1\beta_2$, and $r_{24} = \alpha_2\alpha_4 + \beta_2\beta_4$.

Since

$$\alpha_1 = \frac{c_1\sigma_a}{\sigma_1}$$

and $$\alpha_4 = \frac{c_1\sigma_a}{\sigma_4}$$

therefore $$\alpha_4 = \alpha_1 \frac{\sigma_1}{\sigma_4}.$$

Similarly $$\beta_4 = \beta_1 \frac{\sigma_1}{\sigma_4}.$$

This yields

$$r_{24} = (\alpha_1\alpha_2 + \beta_1\beta_2) \frac{\sigma_1}{\sigma_4} = r_{12} \frac{\sigma_1}{\sigma_4} \ldots\ldots [21]$$

Similarly

$$r_{34} = r_{13} \frac{\sigma_1}{\sigma_4} \ldots\ldots\ldots\ldots [\textit{See } 21]$$

Substituting values for r_{24} and r_{34} just found we obtain

$$\frac{\sigma^2{}_4}{\sigma^2{}_1} = \frac{r^2{}_{12} + r^2{}_{13} - 2r_{12}\, r_{13}\, r_{23}}{1 - r^2{}_{23}} \ldots\ldots\ldots [22]$$

Since σ_4/σ_1 must lie between 0 and 1 the required criterion is

$$0 < \frac{r^2{}_{12} + r^2{}_{13} - 2r_{12}\, r_{13}\, r_{23}}{1 - r^2{}_{23}} < 1 \ldots\ldots\ldots [23]$$

As the denominator is always positive the expression can be less than the lower limit only in case

$$|2r_{12}\, r_{13}\, r_{23}| > |r^2{}_{12} + r^2{}_{13}|$$

This is never possible for

$$|r^2_{12} + r^2_{13}| \geqq |2r_{12} r_{13}| \geqq |2r_{12} r_{13} r_{23}|$$

Now let us consider the upper limit. For the expression to exceed the upper limit is equivalent to the following statement.

$$\frac{r^2_{12} + r^2_{13} - 2r_{12} r_{13} r_{23}}{1 - r^2_{23}} - 1 > 0$$

This in turn can only hold if

$$r^2_{12} + r^2_{13} + r^2_{23} - 2r_{12} r_{13} r_{23} - 1 > 0$$

But Yule has shown (1912) that this is never possible. Accordingly we have proved Proposition 7. The next two propositions, of course, follow without proof.

PROPOSITION 8.—*Three variables may always be thought of as due to two general factors plus two specific factors.*

PROPOSITION 9.—*Three variables may always be thought of as due to two general factors plus three specific factors.*

If specific factors are present, and they always are even if they constitute nothing more than the chance elements entering into a score, the need for more than two general factors can never be established by the study of three variables only. This finding is of importance as it defines our study as a four-or-more-variable investigation. The crux of the study devolves upon propositions dealing with four, with five, and with a still larger number of variables. The basic relationship used by Spearman and first devised by him is that given in the next proposition.

PROPOSITION 10.—*Four variables may be thought of as due to one general factor plus four specific factors when*

$$r_{12} r_{34} = r_{13} r_{24} = r_{14} r_{23} \quad \ldots\ldots\ldots\ldots [24]$$

Another way of expressing this is to say that

$$\left.\begin{aligned} \frac{r_{12}}{r_{13}} &= \frac{r_{24}}{r_{34}} \\ \frac{r_{12}}{r_{14}} &= \frac{r_{23}}{r_{34}} \\ \frac{r_{13}}{r_{14}} &= \frac{r_{23}}{r_{24}} \end{aligned}\right\} \ldots\ldots\ldots\ldots [25]$$

Still another way is to say that

$$\left.\begin{aligned} r_{12}\,r_{34} - r_{13}\,r_{24} &= 0 \\ r_{12}\,r_{34} - r_{14}\,r_{23} &= 0 \\ r_{13}\,r_{24} - r_{14}\,r_{23} &= 0 \end{aligned}\right\} \ldots\ldots\ldots [26]$$

Spearman, who first derived these equations, has called such differences as these "tetrad differences." The term is used throughout the present treatment and the following abridged notation employed:

$$\left.\begin{aligned} t_{1\,2\,3\,4} &= r_{12}\,r_{34} - r_{13}\,r_{24} \\ t_{1\,2\,4\,3} &= r_{12}\,r_{34} - r_{14}\,r_{23} \\ t_{1\,3\,4\,2} &= r_{13}\,r_{24} - r_{14}\,r_{23} \\ t_{1\,3\,2\,4} &= r_{13}\,r_{24} - r_{12}\,r_{34} \\ t_{1\,4\,2\,3} &= r_{14}\,r_{23} - r_{12}\,r_{34} \\ t_{1\,4\,3\,2} &= r_{14}\,r_{23} - r_{13}\,r_{24} \end{aligned}\right\} \ldots\ldots [\textit{See } 1]$$

The last three tetrads are merely the first three with the signs reversed. Only two of the six are independent, so that there are two conditions to be met. The reader should note that the third subscript of a t is the second subscript of the third r appearing in the tetrad difference. Having noted this the entire tetrad difference may be written down. We will now prove Proposition 10.

Let

$$\left.\begin{aligned} x_1 &= c_1a + e_1 \\ x_2 &= c_2a + e_2 \\ x_3 &= c_3a + e_3 \\ x_4 &= c_4a + e_4 \end{aligned}\right\} \ldots\ldots\ldots\ldots [\textit{See } 10]$$

Defining α_1, α_2 . . . . as before, it readily follows that:

$$\left.\begin{aligned} r_{12} &= \alpha_1\,\alpha_2 \\ r_{13} &= \alpha_1\,\alpha_3 \\ r_{14} &= \alpha_1\,\alpha_4 \\ r_{23} &= \alpha_2\,\alpha_3 \\ r_{24} &= \alpha_2\,\alpha_4 \\ r_{34} &= \alpha_3\,\alpha_4 \end{aligned}\right\} \ldots\ldots\ldots\ldots [\textit{See } 11]$$

From this we immediately obtain:

$$r_{12}\,r_{34} = r_{13}\,r_{24} = r_{14}\,r_{23} = \alpha_1\,\alpha_2\,\alpha_3\,\alpha_4 \ldots\ldots\ldots [27]$$

This establishes the sufficiency of the equality of the tetrad differences with 0 to enable one to think of four variables as being consequent to one general factor plus four specific factors. We may at this point state the converse proposition. *If a tetrad difference does not equal 0, two or more general factors plus specific factors are necessary to account for the inter-correlations between the four variables.* This converse proposition is almost certainly true so long as linear regression only is involved.

We must now obtain a formula giving the probable error of the tetrad difference. The writer has used since 1922 a formula yielding the standard error of the tetrad quotient, $r_{12}\,r_{34}/r_{13}\,r_{24}$, but he has recently been converted to the use of tetrad differences instead (in part because Spearman uses them) and he has derived a standard error formula for the same. Spearman and Holzinger (1925) also have a formula for the probable error of a tetrad difference, but as their formula gives the probable error in case the true difference is equal to 0 the writer prefers his own slightly longer formula, for the latter does not depend upon the true value of the difference being equal to 0. In passing, it may be stated that the writer's formula reduces to that of Spearman and Holzinger's when the true tetrad difference does equal 0, so that there is no inconsistency between the two formulas. We have:

$$t_{1234} = r_{12}\, r_{34} - r_{13}\, r_{24}$$
$$t_{1234} + \Delta_{1234} = (r_{12} + \Delta_{12})\,(r_{34} + \Delta_{34}) - (r_{13} + \Delta_{13})\,(r_{24} + \Delta_{24})$$

Expanding and subtracting t_{1234} from each member and neglecting second degree terms in the Δ's, since they are of an order $1/\sqrt{N}$ to the first degree terms and thus small if N is appreciable, we have

$$\Delta_{1234} = \Delta_{12} r_{34} + \Delta_{34} r_{12} - \Delta_{13} r_{24} - \Delta_{24} r_{13}$$

Squaring, summing, and dividing by the population yields

$$\sigma^2_{t_{1234}} = \sigma^2_{r_{12}} r^2_{34} + \sigma^2_{r_{34}} r^2_{12} + \sigma^2_{r_{13}} r^2_{24} + \sigma^2_{r_{24}} r^2_{13}$$
$$+ 2\,(\sigma_{r_{12}} r_{34}\, \sigma_{r_{34}} r_{12}\, r_{r_{12} r_{34}} + \sigma_{r_{13}} r_{24} \sigma_{r_{24}} r_{13}\, r_{r_{13} r_{24}} - \sigma_{r_{12}} r_{34} \sigma_{r_{13}} r_{24} r_{r_{12} r_{13}}$$
$$- \sigma_{r_{12}} r_{34} \sigma_{r_{24}} r_{13} r_{r_{12} r_{24}} - \sigma_{r_{34}} r_{12}\, \sigma_{r_{13}} r_{24}\, r_{r_{13} r_{34}} - \sigma_{r_{34}} r_{12}\, \sigma_{r_{24}} r_{13}\, r_{r_{24} r_{34}})$$

Utilizing Filon and Pearson's formulas, numbers 127 and 128 of Kelley, *Statistical Method,* for the correlation between correlation coefficients and the usual formula $(1 - r^2)/\sqrt{N}$ for the standard error of a correlation coefficient enables one to reduce the preceding to

$$\sigma_{t_{1234}} = \frac{1}{\sqrt{N}} [r^2_{12} + r^2_{13} + r^2_{24} + r^2_{34} + 2r_{12}\, r_{14}\, r_{23}\, r_{34}$$
$$+ 2r_{13}\, r_{14}\, r_{23}\, r_{24} - 2r_{12}\, r_{13}\, r_{23} - 2r_{12}\, r_{14}\, r_{24} - 2r_{13}\, r_{14}\, r_{34} - 2r_{23}\, r_{24}\, r_{34}$$
$$+ t^2_{1234}(r^2_{12} + r^2_{13} + r^2_{14} + r^2_{23} + r^2_{24} + r^2_{34} - 4)]^{\frac{1}{2}} \ldots\ldots\ldots [28]$$

Though a number of abridgments to this value of $\sigma_{t_{1234}}$ have been investigated, none give close enough approximations to this full value to recommend.

Proposition 11.—*Four variables may be thought of as due to one general factor plus three specific factors when the product of the three inter-correlations involving three of the variables is equal to the square of the product of the other three inter-correlations. (A more accurate statement of the sufficient condition is found at the end of the next paragraph.)*

When four variables give two tetrads which are each equal to 0 within the limits of their probable errors, it may then be desirable to ascertain what is the nature of the one common function. The four variables each possess this common factor plus four specific factors, so that by studying variable 1, which, let us say, is a certain psychological test, one does not know whether the mental tasks which he finds involved therein are representative of the common factor or of the specific factor. He may, however, obtain much light upon this if the score on a second similar form of this Test 1 is available. On page 38 we let $x_1 = c_1\, a + e_1$, wherein e_1 was a factor specific to Test 1. Let us here divide e_1 into two parts, $e_1 = b + d$, in which b is a factor specific to Test 1 but not chance, and d is the chance factor specific to Test 1, then $x_1 = c_1\, a + b + d$. For the second similar form of this test we have

$$x_I = c_1\, a + b + D$$

in which $c_1\, a$ and b are identical with $c_1\, a$ and b in form 1, but in which for any individual D is a chance factor entirely unrelated to the chance factor d. If we correct all correlations involving variable 1 for attenuation (for variable 1 only) we eliminate the systematic effect of the chance factor d but not the effect of the specific factor b and thus the correlations corrected for attenuation (for variable 1 only) are those between the following sets of variables. The quantity $c_1\, a + b$ becomes the first variable and this we will designate by x_∞; the second variable is x_2, the third is x_3, and the fourth is x_4. The correlations are $r_{\infty 2}$, $r_{\infty 3}$, $r_{\infty 4}$, r_{23}, r_{24}, r_{34}. If

x_1 is such that it has no factor b, being in truth composed merely of the general factor c_1a and a chance factor, we could then, of course, carefully study Test 1, attempt to eliminate from our concept of it such features as are chance, and secure a residuum that is $c_1 a$ and nothing else. If there is no factor b entering into Test 1 we may write $x_1 = c_1a + d$ and $x_\infty = c_1a$, using x_∞ in place of x_1 and paralleling the steps involved in the proof of the last proposition we obtain

$$x_\infty = c_1a$$

$$x_2 = c_2a + e_2$$

$$x_3 = c_3a + e_3$$

$$x_4 = c_4a + e_4$$

$$r_{\infty 2} = \alpha_2 \qquad r_{23} = \alpha_2\alpha_3$$

$$r_{\infty 3} = \alpha_3 \qquad r_{24} = \alpha_2\alpha_4$$

$$r_{\infty 4} = \alpha_4 \qquad r_{34} = \alpha_3\alpha_4$$

$$\left.\begin{aligned} r_{\infty 2}\, r_{\infty 3} &= r_{23} \\ r_{\infty 2}\, r_{\infty 4} &= r_{24} \\ r_{\infty 3}\, r_{\infty 4} &= r_{34} \end{aligned}\right\} \qquad [29]$$

$$r^2_{\infty 2}\, r^2_{\infty 3}\, r^2_{\infty 4} = r_{23}\, r_{24}\, r_{34} \qquad [30]$$

If Equations 29 and thus as a consequence Equation 30 hold, we know that all of Test 1 except that part of it that is chance is the general factor. The formulas giving the probable errors of $(r_{\infty 2}\, r_{\infty 3} - r_{23})$, $(r_{\infty 2}\, r_{\infty 4} - r_{24})$, $(r_{\infty 3}\, r_{\infty 4} - r_{34})$, and $(r^2_{\infty 2}\, r^2_{\infty 3}\, r^2_{\infty 4} - r_{23}\, r_{24}\, r_{34})$ have not been derived so that this criterion is not of immediate practical value. The relationship stated in Formula 30 is a necessary but not a sufficient condition. The sufficient requirement is that the three equations [29] hold. It is conceivable that Formula 30 might hold without all three of these holding, though this is improbable in view of the likelihood of the left-hand member falling short and not exceeding the right-hand member in each of the equations in [29].

Proposition 12.—*Four variables may be thought of as due to two general factors and four specific factors when a value* $\alpha^2_1 \geqq 0$ *and* $\leqq 1$ *can be found such that*

$$0 \leqq \frac{(\alpha^2_1 r_{24} - r_{12} r_{14})\,(\alpha^2_1 r_{23} - r_{12} r_{13})}{\alpha^2_1\,(\alpha^2_1 r_{34} - r_{13} r_{14})} \leqq 1 \ldots\ldots [31]$$

and when every similar function obtained by permuting the variables is satisfied.

Let

$$\left.\begin{aligned} x_1 &= C_1A + K_1B + e_1 \\ x_2 &= C_2A + K_2B + e_2 \\ x_3 &= C_3A + K_3B + e_3 \\ x_4 &= C_4A + K_4B + e_4 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots [32]$$

in which, as before, $\sigma_A = \sigma_B = 1$ and A, B, e_1, e_2, e_3, and e_4 are uncorrelated. In place of the variables A and B it will prove convenient to rotate the axes using new variables a and b defined by the equations

$$\begin{aligned} A &= a \cos \vartheta - b \sin \vartheta \\ B &= a \sin \vartheta + b \cos \vartheta \end{aligned}$$

The new variables a and b are uncorrelated as were by hypothesis A and B, for we have

$$\begin{aligned} a &= A \cos \vartheta + B \sin \vartheta \\ b &= -A \sin \vartheta + B \cos \vartheta \end{aligned}$$

$$\begin{aligned} \Sigma ab = \Sigma(A \cos \vartheta + B \sin \vartheta)\,(-A \sin \vartheta + B \cos \vartheta) = \\ - \sin \vartheta \cos \vartheta\, \Sigma A^2 + \cos^2 \vartheta\, \Sigma AB \\ - \sin^2 \vartheta\, \Sigma AB + \sin \vartheta \cos \vartheta\, \Sigma B^2 \end{aligned}$$

which equals zero since the $\Sigma AB = 0$ and since $\Sigma A^2 = \Sigma B^2$. Making the indicated substitutions for A and B we have

$$x_1 = C_1 a \cos\vartheta - C_1 b \sin\vartheta + K_1 a \sin\vartheta + K_1 b \cos\vartheta + e_1$$

$$= (C_1 \cos\vartheta + K_1 \sin\vartheta)\, a + (K_1 \cos\vartheta - C_1 \sin\vartheta)\, b + e_1$$

Since there is no limitation upon ϑ let us choose such a value of ϑ that

$$K_1 \cos\vartheta - C_1 \sin\vartheta = 0$$

and let us designate $(C_1 \cos\vartheta + K_1 \sin\vartheta)$ by the symbol c_1. Then we have

$$x_1 = c_1 a + e_1 \quad \ldots\ldots\ldots\ldots\ldots [\textit{See } 10]$$

Making the same substitution in the equations giving x_2, x_3, and x_4 and designating the constants which multiply the two variable general factors by lower-case letters we have

$$\left.\begin{aligned} x_2 &= c_2 a + k_2 b + e_2 \\ x_3 &= c_3 a + k_3 b + e_3 \\ x_4 &= c_4 a + k_4 b + e_4 \end{aligned}\right\} \ldots\ldots\ldots [\textit{See } 32]$$

This set of four equations in which $k_1 = 0$ defines x_1, x_2, x_3, and x_4 with as much generality as the first set of four. We may state the following proposition, derived incidentally in our proof of Proposition 12.

Proposition 13.—*If it is possible to express four variables as due to two general factors plus four specific factors, it is always possible to express the same four variables as due to two general factors one of which runs through all four of the variables, while the other one runs through but three of them, plus four specific factors.*

Let us call

$$\frac{c_1\sigma_a}{\sigma_1} = \alpha_1 ; \frac{c_2\sigma_a}{\sigma_2} = \alpha_2 ; \frac{c_3\sigma_a}{\sigma_3} = \alpha_3 ; \frac{c_4\sigma_a}{\sigma_4} = \alpha_4 \ldots . [\textit{See } 12]$$

$$\frac{k_2\sigma_b}{\sigma_2} = \beta_2 ; \frac{k_3\sigma_b}{\sigma_3} = \beta_3 ; \frac{k_4\sigma_b}{\sigma_4} = \beta_4 \ldots\ldots\ldots . [\textit{See } 20]$$

$$\left.\begin{aligned} r_{12} &= \alpha_1 \alpha_2 \\ r_{13} &= \alpha_1 \alpha_3 \\ r_{14} &= \alpha_1 \alpha_4 \end{aligned}\right\} \ldots\ldots\ldots\ldots [33]$$

$$\left.\begin{aligned} r_{23} &= \alpha_2 \alpha_3 + \beta_2 \beta_3 \\ r_{24} &= \alpha_2 \alpha_4 + \beta_2 \beta_4 \\ r_{34} &= \alpha_3 \alpha_4 + \beta_3 \beta_4 \end{aligned}\right\} \ldots\ldots\ldots\ldots [34]$$

Eliminating α_2, α_3, α_4, β_3, and β_4 from these six equations we obtain

$$\frac{(\alpha^2{}_1 r_{23} - r_{12} r_{13})\ (\alpha^2{}_1 r_{24} - r_{12} r_{14})}{\alpha^2{}_1\ (\alpha^2{}_1 r_{34} - r_{13} r_{14})} = \beta^2{}_2 \ldots\ldots [35]$$

There are two other equations of this type involving in the right-hand members $\beta^2{}_3$ and $\beta^2{}_4$ and the same $\alpha^2{}_1$ in their left-hand members. There are four such sets, giving 12 conditions in all to be met. Since $\alpha^2{}_1$ and $\beta^2{}_2$ are independent and since each lies between 0 and 1 we have the proposition as stated.

Dr. Harold Hotelling has kindly provided the following set of necessary conditions which are more readily investigated than are the 12 sufficient equations in Formula 35.

Substituting the values α_2, α_3, and α_4 obtained from the three equations [33] into the three [34], yields

$$\beta_2 \beta_3 = r_{23} - \frac{r_{12} r_{13}}{\alpha^2{}_1} \qquad \beta_2 \beta_4 = r_{24} - \frac{r_{12} r_{14}}{\alpha^2{}_1}$$

$$\beta_3 \beta_4 = r_{34} - \frac{r_{13} r_{14}}{\alpha^2{}_1}$$

Multiplying these three equations member by member and writing z for $1/\alpha^2_1$ we obtain

$$\beta^2_2\,\beta^2_3\,\beta^2_4 = (r_{23} - r_{12}\,r_{13}\,z)\ \ (r_{24} - r_{12}\,r_{14}\,z)\ \ (r_{34} - r_{13}\,r_{14}\,z)$$
$$= f(z) \qquad [36]$$

If the assumption of the adequacy of two general factors is correct then there must be a value for z greater than 1 which will give a value of $f(z)$ lying between zero and 1 inclusive. The coefficient of z_3 is negative, consequently if the curve $y = f(z)$ enters the strip $z \geqq 1$, $0 \leqq y \leqq 1$, the equation $f(z) = 0$ must have a real root greater than 1. But the three roots of this equation are

$$\frac{r_{23}}{r_{12}\,r_{13}},\quad \frac{r_{24}}{r_{12}\,r_{14}},\quad \frac{r_{34}}{r_{13}\,r_{14}}.$$

If all of these are less than 1 the two-factor hypothesis is untenable. Three other necessary conditions of consistency are obtained by permuting the subscript 1 in turn with 2, 3, and 4. Thus for consistency one at least of the three quantities in each of the following four sets must be greater than or equal to unity.

$$\left.\begin{matrix} \dfrac{r_{24}}{r_{14}\,r_{12}} & \dfrac{r_{34}}{r_{14}\,r_{13}} & \dfrac{r_{23}}{r_{12}\,r_{13}} \\[2ex] \dfrac{r_{14}}{r_{12}\,r_{24}} & \dfrac{r_{13}}{r_{12}\,r_{23}} & \dfrac{r_{34}}{r_{23}\,r_{24}} \\[2ex] \dfrac{r_{12}}{r_{13}\,r_{23}} & \dfrac{r_{14}}{r_{13}\,r_{34}} & \dfrac{r_{24}}{r_{23}\,r_{34}} \\[2ex] \dfrac{r_{12}}{r_{14}\,r_{24}} & \dfrac{r_{13}}{r_{14}\,r_{34}} & \dfrac{r_{23}}{r_{24}\,r_{34}} \end{matrix}\right\} \qquad [37]$$

The foregoing are necessary but not sufficient conditions. If these necessary conditions are met, one should then investigate the sufficient conditions in Formula 35 to finally establish the adequacy of two general factors.

Let us here present a problem by stating a proposition.

PROPOSITION 14.—*If four variables taken three at a time are such that any three may be thought of as due to one general factor plus specific factors, then the four variables may always be thought of as due to two general factors plus specific factors.*

The writer has found no ready means of either proving or disproving this. Though considerable theoretical interest attaches to it, we do not need it for our further treatment and will therefore leave it as an unsolved problem.

In the search for variables requiring more than one general factor we can start with three variables. If variables taken three at a time reveal no such need for more than one general factor, then we will investigate variables four at a time with reference to the adequacy of one general factor and when variables four at a time cannot be adequately represented by one general factor we will investigate variables five at a time with reference to the sufficiency of two general factors. In other words Proposition 12 seldom needs to be an instrument in the investigation.

There are occasionally, however, situations in which its use is valuable. If four variables taken three at a time reveal two sets of three which cannot be thought of as involving one general factor, the question arises as to whether the two general factors demanded for x_1, x_2, x_3 are the same as the two required for x_1, x_2, x_4. We may illustrate such a situation as this by the data of Table II. The

TABLE II

TABLE OF INTER-CORRELATIONS

	x_1	x_2	x_3
x_2..........	.050		
x_3..........	.220	.291	
x_4..........	.025	.069	—.087

variables have been chosen because from evidence at hand there is known to be much independence between them. The data are for 109 children in the seventh grade.

x_1 = Score on a Speed of Reading Test
x_2 = Score on a Power in Arithmetic Reasoning Test
x_3 = Score on a Manipulation of Spatial Relationships Test
x_4 = Score on an Interest in Physical Activity Test

Examining variables x_1, x_2, x_3, we find $r_{13} r_{23} > r_{12}$. Accordingly, Proposition 5 does not hold for these three variables. Also $r_{23} r_{24} > r_{34}$. Thus again Proposition 5 does not hold. For variables x_1, x_2, x_4 Proposition 5 maintains. Can the two factors required for variables x_1, x_2, x_3 also be thought of as the two required for x_2, x_3, x_4? We will first investigate the necessary conditions [37].

$\frac{r_{24}}{r_{14} r_{12}} > 1$	$\frac{r_{34}}{r_{14} r_{13}} < 1$	$\frac{r_{23}}{r_{12} r_{13}} > 1$
$\frac{r_{14}}{r_{12} r_{24}} > 1$	$\frac{r_{13}}{r_{12} r_{23}} > 1$	$\frac{r_{34}}{r_{23} r_{24}} < 1$
$\frac{r_{12}}{r_{13} r_{23}} = .78 < 1$	$\frac{r_{14}}{r_{13} r_{34}} < 1$	$\frac{r_{24}}{r_{23} r_{34}} < 1$
$\frac{r_{12}}{r_{14} r_{24}} > 1$	$\frac{r_{13}}{r_{14} r_{34}} < 1$	$\frac{r_{23}}{r_{24} r_{34}} < 1$

In the third set no one of the three quotients is greater than unity, so two general factors plus specific factors will not suffice to explain these four variables. Though in this case it is not possible to express four variables as due to two general factors plus four specific factors, generally the criteria just investigated are very easily satisfied, so easily, in fact, that the writer has not made a serious search for four variables which cannot be represented as arising from two general factors plus four specific factors. He has extensively studied the inter-correlations between five variables where much less freedom is present. The probable error of the four-variable criterion [35] has not been investigated, but as the separate functions [37] are the reciprocals of [15] the standard error of one of these functions is given by [16] after first replacing f by $1/f$.

PROPOSITION 15.—*Five variables may be thought of as due to two general factors plus five specific factors when the following function, which will be referred to as the pentad criterion, because five variables are involved, equals zero*:

$$\begin{aligned} f_{12345} = {} & r_{12} r_{13} r_{24} r_{35} r_{45} + r_{14} r_{15} r_{23} r_{24} r_{35} + r_{12} r_{14} r_{25} r_{34} r_{35} \\ & + r_{13} r_{15} r_{24} r_{25} r_{34} + r_{12} r_{15} r_{23} r_{34} r_{45} + r_{13} r_{14} r_{23} r_{25} r_{45} \\ & - r_{12} r_{14} r_{23} r_{35} r_{45} - r_{13} r_{15} r_{23} r_{24} r_{45} - r_{12} r_{15} r_{24} r_{34} r_{35} \\ & - r_{13} r_{14} r_{24} r_{25} r_{35} - r_{12} r_{13} r_{25} r_{34} r_{45} - r_{14} r_{15} r_{23} r_{25} r_{34} \quad [38] \end{aligned}$$

This is a basic criterion in the study of differentiable abilities. The proof of it, which follows, utilizes the same notation as the proofs of the earlier propositions, with such obvious extensions as are required, due to a larger number of variables, and with such additions as will be noted. Let

$$\left.\begin{aligned} x_1 &= c_1 a + k_1 b + e_1 \\ x_2 &= c_2 a + k_2 b + e_2 \\ x_3 &= c_3 a + k_3 b + e_3 \\ x_4 &= c_4 a + k_4 b + e_4 \\ x_5 &= c_5 a + k_5 b + e_5 \end{aligned}\right\} \quad [\textit{See } 32]$$

then

$$\left.\begin{aligned} r_{12} &= \alpha_1 \alpha_2 + \beta_1 \beta_2 \\ r_{13} &= \alpha_1 \alpha_3 + \beta_1 \beta_3 \\ r_{14} &= \alpha_1 \alpha_4 + \beta_1 \beta_4 \\ r_{15} &= \alpha_1 \alpha_5 + \beta_1 \beta_5 \\ r_{23} &= \alpha_2 \alpha_3 + \beta_2 \beta_3 \\ r_{24} &= \alpha_2 \alpha_4 + \beta_2 \beta_4 \\ r_{25} &= \alpha_2 \alpha_5 + \beta_2 \beta_5 \\ r_{34} &= \alpha_3 \alpha_4 + \beta_3 \beta_4 \\ r_{35} &= \alpha_3 \alpha_5 + \beta_3 \beta_5 \\ r_{45} &= \alpha_4 \alpha_5 + \beta_4 \beta_5 \end{aligned}\right\} \quad [\textit{See } 34]$$

Let $\beta_1/\alpha_1 = \lambda_1$; $\beta_2/\alpha_2 = \lambda_2$; etc. Then

$$\frac{r_{12}}{1+\lambda_1\lambda_2} = \alpha_1\alpha_2 \quad \text{[i]}$$

$$\frac{r_{13}}{1+\lambda_1\lambda_3} = \alpha_1\alpha_3 \quad \text{[ii]}$$

$$\frac{r_{14}}{1+\lambda_1\lambda_4} = \alpha_1\alpha_4 \quad \text{[iii]}$$

$$\frac{r_{15}}{1+\lambda_1\lambda_5} = \alpha_1\alpha_5 \quad \text{[iv]}$$

$$\frac{r_{23}}{1+\lambda_2\lambda_3} = \alpha_2\alpha_3 \quad \text{[v]}$$

$$\frac{r_{24}}{1+\lambda_2\lambda_4} = \alpha_2\alpha_4 \quad \text{[vi]}$$

$$\frac{r_{25}}{1+\lambda_2\lambda_5} = \alpha_2\alpha_5 \quad \text{[vii]}$$

$$\frac{r_{34}}{1+\lambda_3\lambda_4} = \alpha_3\alpha_4 \quad \text{[viii]}$$

$$\frac{r_{35}}{1+\lambda_3\lambda_5} = \alpha_3\alpha_5 \quad \text{[ix]}$$

$$\frac{r_{45}}{1+\lambda_4\lambda_5} = \alpha_4\alpha_5 \quad \text{[x]}$$

From Equations i and viii, from ii and vi, and from iii and v we obtain

$$\frac{r_{12}\,r_{34}}{(1+\lambda_1\lambda_2)\,(1+\lambda_3\lambda_4)} = \frac{r_{13}\,r_{24}}{(1+\lambda_1\lambda_3)\,(1+\lambda_2\lambda_4)}$$

$$= \frac{r_{14}\,r_{23}}{(1+\lambda_1\lambda_4)\,(1+\lambda_2\lambda_3)}$$

This leads to

$$\frac{r_{12}\,r_{34} - r_{14}\,r_{23}}{\lambda_1\lambda_2 + \lambda_3\lambda_4 - \lambda_1\lambda_4 - \lambda_2\lambda_3} = \frac{r_{12}\,r_{34} - r_{13}\,r_{24}}{\lambda_1\lambda_2 + \lambda_3\lambda_4 - \lambda_1\lambda_3 - \lambda_2\lambda_4}$$

which may be written

$$\frac{r_{12}r_{34}-r_{14}r_{23}}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}=\frac{r_{12}r_{34}-r_{13}r_{24}}{(\lambda_1-\lambda_4)(\lambda_2-\lambda_3)} \ldots\ldots [xi]$$

Similarly

$$\frac{r_{12}r_{35}-r_{13}r_{25}}{(\lambda_1-\lambda_5)(\lambda_2-\lambda_3)}=\frac{r_{12}r_{35}-r_{15}r_{23}}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_5)} \ldots\ldots [xii]$$

$$\frac{r_{12}r_{45}-r_{15}r_{24}}{(\lambda_1-\lambda_4)(\lambda_2-\lambda_5)}=\frac{r_{12}r_{45}-r_{14}r_{25}}{(\lambda_1-\lambda_5)(\lambda_2-\lambda_4)} \ldots\ldots [xiii]$$

Multiplying [xi], [xii], and [xiii] we obtain

$$(r_{12}r_{34}-r_{14}r_{23})(r_{12}r_{35}-r_{13}r_{25})(r_{12}r_{45}-r_{15}r_{24})$$
$$=(r_{12}r_{34}-r_{13}r_{24})(r_{12}r_{35}-r_{15}r_{23})(r_{12}r_{45}-r_{14}r_{25}) \ldots [39]$$[1]

Multiplying and factoring, Equation 39 may be written

$$(r_{12}r_{13}r_{45}+r_{14}r_{15}r_{23})(r_{24}r_{35}-r_{25}r_{34})$$
$$+(r_{12}r_{14}r_{35}+r_{13}r_{15}r_{24})(r_{25}r_{34}-r_{23}r_{45})$$
$$+(r_{12}r_{15}r_{34}+r_{13}r_{14}r_{25})(r_{23}r_{45}-r_{24}r_{35})=0 \ldots\ldots [40]$$

It can also be written as the sum of six terms minus the sum of six others by multiplying out. This latter way has seemed the most convenient, and to abridge the notation the following symbol has been used: $\pi{:}12{:}13{:}24{:}35{:}45 = r_{12}r_{13}r_{24}r_{35}r_{45}$ and similarly for π's with the other following numbers. With this notation [40] may be written

$$f_{12345}=\pi\,12{:}13{:}24{:}35{:}45+\pi\,14{:}15{:}23{:}24{:}35+\pi\,12{:}14{:}25{:}34{:}35$$
$$+\pi\,13{:}15{:}24{:}25{:}34+\pi\,12{:}15{:}23{:}34{:}45+\pi\,13{:}14{:}23{:}25{:}45$$
$$-\pi\,12{:}14{:}23{:}35{:}45-\pi\,13{:}15{:}23{:}24{:}45-\pi\,12{:}15{:}24{:}34{:}35$$
$$-\pi\,13{:}14{:}24{:}25{:}35-\pi\,12{:}13{:}25{:}34{:}45-\pi\,14{:}15{:}23{:}25{:}34$$

[*See* 38]

[1] I am indebted to Dr. C. E. Rhodes for assistance in the solution here given. He is responsible for the steps represented by Equations xi, xii, xiii, and 39.

Let us now return to the proposition dealing with four variables. Suppose we have four variables, x_1, x_2, x_3, x_4, which we wish to investigate with reference to the adequacy of two general factors. Let us add a fifth variable, x_5, which is entirely chance. Then $r_{15} = r_{25} = r_{35} = r_{45} = 0$. Reference to [40] shows that in this case the pentad function equals zero, so that on the basis of this we might be inclined to conclude that any four variables plus a fifth which was chance could be represented by two general factors plus specific factors. If this conclusion is sound, then it follows that any four variables could always be represented by two general factors plus specific factors. The conclusion, however, is not sound, for the reason that we are not concerned with the absolute value of f_{12345}, but with it in comparison with its standard or probable error. It can be shown that in general if x_5 is chance, then

$$\frac{f_{12345}}{\sigma_{f_{12345}}} = \frac{0}{0}$$

so that we have an indeterminate ratio. Three conclusions may be drawn from the situation. First, we must not conclude that every four variables can be thought of as arising from two general factors plus specific factors. Second, we must in all cases draw our conclusions from $f_{12345}/\sigma_{f_{12345}}$ and not from f_{12345} alone. Third, if five variables, none of which is chance, are such that some four of them meet conditions [26] then without further test it is known that the five variables may be thought of as arising from not over two general factors plus specific factors. Accordingly, we will only use this pentad criterion [40] where the earlier use of the tetrad criterion [26] has shown that no four selected from five different variables meet criterion [26].

A knowledge of the standard error of f_{12345} is essential, and though its calculation is very laborious it must be undertaken. Many approximations have been attempted, but none is believed trustworthy, so that the full formula for the standard error of f_{12345} is recommended. The expression of this formula in terms of N, the population, and the correlation coefficients is unmanageable, and so far as the writer has been able to discover gives no promise of greatly simplifying, so that a formula giving $\sigma_{f_{12345}}$ in the primitive or original constants, N, r_{12}, r_{13}, r_{14}, r_{15}, r_{23}, r_{24}, r_{25}, r_{34}, r_{35}, r_{45}, is not attempted. The numerical solution for any given problem is, however, entirely feasible. Certain new symbols are used in the following derivation. Their meanings are given immediately following the final formula obtained. We will start with f_{12345} as given in [38].

$$\Delta f = \Delta_{12} C_{12} + \Delta_{13} C_{13} + \Delta_{14} C_{14} + \Delta_{15} C_{15} + \Delta_{23} C_{23} + \Delta_{24} C_{24} + \Delta_{25} C_{25} + \Delta_{34} C_{34} + \Delta_{35} C_{35} + \Delta_{45} C_{45} \ldots\ldots [41]$$

Squaring, summing, and dividing by the population gives

$$\sigma^2_f = \frac{1}{N} [S(1 - r^2_{12})^2 C^2_{12} + SC_{12} C_{13} p_{12:13} + SC_{12} C_{34} p_{12:34}] \quad [42]$$

When the polynomial giving Δf is squared and the several terms summed and divided by the population, three types of terms appear. There are ten terms of the sort $\sigma^2_{r_{12}} C^2_{12}$

in which

$$\sigma^2_{r_{12}} = \frac{1}{N} (1 - r^2_{12})^2$$

and in which

$$C_{12} = \frac{\text{(the sum of all terms in [38] containing } r_{12})}{r_{12}} \ldots\ldots [43]$$

and C's with other subscripts have similar meanings. The term in [42] designated $S(1 - r^2)^2 C^2_{12}$ stands for the sum of these ten terms, except for the factor $1/N$ which is taken care of separately.

There are 60 terms, but only 30 different, as each occurs twice, of the sort $\sigma_{r_{12}} C_{12} \sigma_{r_{13}} C_{13} r_{r_{12} r_{13}}$. If the product-moment $\sigma_{r_{12}} \sigma_{r_{13}} r_{r_{12} r_{13}}$ is represented by the notation $p_{12:13}/N$, these 60 terms, except for $1/N$, are represented in [42] by $SC_{12} C_{13} p_{12:13}$. The quantity $p_{12:13}$ depends upon the numerical values of three correlation coefficients r_{12}, r_{13}, r_{23} and may be evaluated by Formula 66.

There are 30 terms, but only 15 different, as each occurs twice, of the sort $\sigma_{r_{12}} C_{12}\, \sigma_{r_{34}}\, C_{34}\, r_{r_{12} r_{34}}$ which if we let $\sigma_{r_{12}}\, \sigma_{r_{34}}\, r_{r_{12} r_{34}} = p_{12:34}/N$ may be represented, except for $1/N$ as in the last term of [42]. The product-moment in this term depends upon the intercorrelations between four variables which are no less than six in number, so that it is scarcely feasible to table values of p for different values of these six correlation coefficients. The following formula has been found convenient for the calculation of $p_{12:34}$:

$$p_{12:34} = N\sigma_{r_{12}}\, \sigma_{r_{34}}\, r_{r_{12} r_{34}}$$

$$= r_{13}\, r_{24} + r_{14}\, r_{23} - r_{12}\, r_{13}\, r_{14} - r_{12}\, r_{23}\, r_{24} - r_{13}\, r_{23}\, r_{34}$$

$$- r_{14}\, r_{24}\, r_{34} + \tfrac{1}{2} r_{12}\, r_{34}\, (r^2_{13} + r^2_{14} + r^2_{23} + r^2_{24}) \ldots\ldots\ [44]$$

It is thus seen that the variance [42] has a total of 100 terms each of which if expanded is made up of a considerable number of terms. As at best it takes considerable time to employ this formula, it has only been used in the present study some ten or a dozen times. It does, however, enable one to obtain a general idea as to the size of the standard error of the pentad functions worked with. An illustration showing all the numerical steps in the computation of σ_f would require many pages and is not here given, but a careful study of the meanings of the symbols just described should enable the student to determine such a standard error when in need of it.

The following five-variable problem is one for which the standard error of the pentad function has been calculated:

$N = 140$ seventh-grade pupils

$x_1 = x_1$ of Table X = score on a Speed of Reading Test

$x_2 = x_2$ of Table X = score on a Reading Power Test

$x_3 = x_3$ of Table X = score on a Speed of Arithmetic Computation Test

$x_4 = x_4$ of Table X = score on an Arithmetic Reasoning Test

$x_5 = x_7$ of Table X = score on a Memory for Meaningful Geometric Forms Test

TABLE III

Table of Inter-Correlations

	x_1	x_2	x_3	x_4
x_2	.6328			
x_3	.2412	—.0553		
x_4	.0586	.0655	.4248	
x_5	.2969	.3322	.0215	.2489

We shall first examine these variables in sets of four to ascertain if any four may be represented by one general factor plus specific factors. The tetrad differences for variables x_1, x_2, x_3, x_4 are

$$t_{1\,2\,3\,4} = (.6328)\ (.4248) - (.2412)\ (.0655) = .25$$

$$t_{1\,2\,4\,3} = (.6328)\ (.4248) - (.0586)\ (-.0553) = .27$$

$$t_{1\,3\,4\,2} = (.2412)\ (.0655) - (.0586)\ (-.0553) = .02$$

We have three tetrad differences involving the same variables. If one were to choose the largest of the three, he would be capitalizing any chance influence that contributed to the making of this difference the largest. Similarly one should not choose the smallest of these three. If we chose the mid-one (in absolute value) there is no systematic chance tendency operating one way or the other. We will thus take the tetrad difference .25 as most representative of the tetrad differences of these four variables.

Following the same procedure for all other combinations of four we determine the data of Table IV.

Before investigating the pentad function it is desirable to know if any of these tetrad differences deviate from 0 by an amount which may readily be attributed to chance. We will therefore calculate the standard error of $t_{1\,5\,4\,3}$, which is the smallest of these five tetrad differences. We find by Formula 42 that the standard error of this difference is .05. Thus the difference is but 1.3 times its standard error. Reference to a table of the normal probability curve would indicate that the chances were about four to one that the difference was significant. However, since the tetrad difference chosen for investigation was so chosen because it was the smallest of the five, this statement of the case is an understatement. The chances are surely appreciably greater than four to one that the difference is significant. We will accordingly consider it probable that no four of these five variables can be thought of as due to one general factor.

TABLE IV

MEDIAN TETRAD DIFFERENCES

Variables	Median Absolute Value of the Three Possible Tetrad Differences	Standard Error of Tetrad Difference
1 2 3 4	$t_{1234} = .25$	
1 2 3 5	$t_{1235} = .07$	
1 2 4 5	$t_{1254} = .14$	
1 3 4 5	$t_{1534} = .07$	.051
2 3 4 5	$t_{2543} = .14$	

Further, since conditions [31] are so readily satisfied, we will not trouble to apply that test but immediately think of two general factors as being sufficient for each set of four variables. Having determined that two general factors are needed for every combination of the five variables taken four at a time, it now becomes of interest to ascertain if the same two can be thought of as underlying every possible set of these five variables taken four at a time. The determination of the pentad criterion and its standard error will enable us to answer this question. The value of the pentad criterion given by [38] is —.0056, and its standard error given by [42] is .0025. Reference to a normal probability table shows that the chances are somewhat less than 3 in 100 that a pentad function as large as this would arise as a matter of chance. We therefore must conclude that not only is a single general factor insufficient to account for these five variables but even two general factors are insufficient. There must be in addition to specific factors no less than three basic independent traits underlying the five variables.

When we have four variables not capable of representation as coming from one general factor plus specific factors, it may be because there is one general factor running through three of the four variables, and (*a*) a second general factor running through three of the four variables, or (*b*) a second general factor running through two of the four variables. By Proposition 13 we need not consider the situation of a second factor running through all four of the variables as by an appropriate choice of the first general factor this can be reduced to a situation in which the second general factor runs through three only of the four variables.

Thus if a certain tetrad difference involving x_1, x_2, x_3, x_4 does not equal 0 we have the two following situations to consider:

First situation

$$\begin{aligned} x_1 &= c_1a + e_1 \\ x_2 &= c_2a + k_2b + e_2 \\ x_3 &= c_3a + k_3b + e_3 \\ x_4 &= c_4a + k_4b + e_4 \end{aligned}$$

Second situation

$$\left.\begin{aligned} x_1 &= c_1a + e_1 \\ x_2 &= c_2a + e_2 \\ x_3 &= c_3a + k_3b + c_3 \\ x_4 &= c_4a + k_4b + c_4 \end{aligned}\right\} \cdots\cdots\cdots\cdots [45]$$

From the first situation we have Equations 33 and 34 already given (page 54), and from the second situation we obtain

$$\left.\begin{aligned} r_{12} &= \alpha_1\,\alpha_2 \\ r_{13} &= \alpha_1\,\alpha_3 \\ r_{14} &= \alpha_1\,\alpha_4 \\ r_{23} &= \alpha_2\,\alpha_3 \\ r_{24} &= \alpha_2\,\alpha_4 \\ r_{34} &= \alpha_3\,\alpha_4 + \beta_3\,\beta_4 \end{aligned}\right\} \cdots\cdots\cdots\cdots [46]$$

In this case

$$\left.\begin{aligned} t_{1234} &= r_{12}r_{34} - r_{13}r_{24} = \alpha_1\alpha_2\beta_3\beta_4 \neq 0 \\ t_{1243} &= r_{12}r_{34} - r_{14}r_{23} = \alpha_1\alpha_2\beta_3\beta_4 \neq 0 \\ t_{1342} &= r_{13}r_{24} - r_{14}r_{23} = t_{1243} - t_{1234} = 0 \end{aligned}\right\} \ldots [47]$$

The characteristic of this second situation is that of the three tetrads two are equal and one equals 0. It is not necessary to state that one of these is equal to 0, for if two of the tetrads are equal, the third is of necessity equal to 0. It so happens that many experimental situations give two nearly equal tetrads which are not equal to 0. As an illustration we may note the following tetrads derived from Table X of chapter iv:

TABLE V

Certain Tetrads for Seventh-Grade Population of 140

Four Variables	Three Tetrads			Designation of Smallest Tetrad	Variables Having Second Factor	
	Largest		Smallest			
1234........	27	25	02	t_{1342}	12 *or*	34
1245........	08	08	01	t_{1452}	12	45
1246........	17	17	00	t_{1462}	12	46
1247........	14	14	00	t_{1472}	12	47
1248........	17	17	00	t_{1482}	12	48
1249........	22	22	00	t_{1492}	12	49
1256........	40	38	01	t_{1652}	12	56
1257........	36	33	03	t_{1752}	12	57
1258........	25	23	01	t_{1852}	12	58
1259........	22	22	01	t_{1952}	12	59
1267........	32	31	01	t_{1762}	12	67
1268........	26	26	00	t_{1862}	12	68
1269........	22	21	01	t_{1692}	12	69
1278........	37	36	01	t_{1782}	12	78
1279........	28	26	02	t_{1792}	12	79
1289........	30	29	01	t_{1892}	12	89
1348........	08	07	02	t_{1834}	14	38
1349........	09	08	01	t_{1394}	14	39
1356........	16	13	03	t_{1563}	13	56
1357........	16	15	01	t_{1753}	13	57
1358........	06	06	00	t_{1583}	13	58
1359........	10	10	00	t_{1953}	13	59

TABLE V—*Continued*

CERTAIN TETRADS FOR SEVENTH-GRADE POPULATION OF 140

Four Variables	Three Tetrads: Largest		Smallest	Designation of Smallest Tetrad	Variables Having Second Factor	
1378........	16	15	01	t_{1783}	13 *or*	78
1379........	13	12	00	t_{1793}	13	79
1389........	13	12	01	t_{1983}	13	89
1479........	08	06	02	t_{1947}	17	49
2345........	14	13	01	t_{2435}	25	34
2346........	14	11	03	t_{2436}	26	34
2347........	15	14	02	t_{2437}	27	34
2348........	12	10	02	t_{2438}	28	34
2349........	14	12	02	t_{2439}	29	34
2458........	06	06	01	t_{2845}	25	48
2459........	09	08	01	t_{2945}	25	49
2569........	07	06	00	t_{2569}	29	56
2578........	06	05	01	t_{2875}	25	78
3456........	29	27	02	t_{3654}	34	56
3457........	28	27	01	t_{3574}	34	57
3458........	19	19	00	t_{3584}	34	58
3459........	18	17	01	t_{3594}	34	59
3467........	26	21	05	t_{3674}	34	67
3468........	20	15	05	t_{3684}	34	68
3478........	29	28	01	t_{3874}	34	78
3479........	22	22	00	t_{3794}	34	79
3489........	22	21	01	t_{3894}	34	89
3567........	14	13	01	t_{3576}	36	57
3568........	08	06	02	t_{3856}	36	58
3569........	08	08	00	t_{3596}	36	59
3678........	15	12	03	t_{3876}	36	78
3679........	11	11	01	t_{3976}	36	79
3689........	11	10	01	t_{3896}	36	89
4567........	10	08	02	t_{4675}	45	67
4578........	09	07	01	t_{4758}	48	57
4579........	17	14	03	t_{4759}	49	57
4789........	12	10	02	t_{4879}	49	78
5678........	20	16	04	t_{5786}	56	78
5679........	11	10	02	t_{5796}	56	79
5689........	19	17	02	t_{5896}	56	89

The cases involving variables x_3, x_4, x_6, x_7; x_3, x_4, x_6, x_8; and x_5, x_6, x_7, x_8 may be doubtful, but omitting these the number and striking character of the remaining sets of tetrads rules out the possibility of chance being the cause. The probable error of the tetrad differences here given is in the neighborhood of .035.

The conditions obtaining between the six correlation coefficients when two of four variables, x_1 and x_2, are dependent upon one general factor a, and the other two, x_3 and x_4, upon two general factors, a and b, are identical with that obtaining if x_3 and x_4 are dependent upon one general factor only and x_1 and x_2 are dependent upon two general factors. Thus if $t_{1\,3\,4\,2} = 0$ (note that this is the tetrad not containing r_{12} or r_{34}), we may conclude that x_1 and x_2 possess some second factor b, or that x_3 and x_4 possess such a second factor, or that x_1 and x_2 possess a second factor b, and x_3 and x_4 a third factor c. We do not know which of these three situations correctly describes the facts and further we never can tell merely from a knowledge of the six inter-correlations, as will now be proved. Let us first note, however, that the third alternative will generally not concern us, as we will systematically choose the hypothesis involving the lesser number of unknown factors.

Proposition 16.—*If the inter-correlations between four variables are such that $t_{1\,2\,3\,4} = t_{1\,2\,4\,3}$ and $t_{1\,3\,4\,2} = 0$, they could conceivably have arisen from four variables x_1, x_2, x_3, x_4 through which was a general factor plus, in addition thereto, a second factor common to x_1 and x_2 or a second factor common to x_3 and x_4.*

If, given the system,

$$\left.\begin{aligned} x_1 &= c_1a + e_1 \\ x_2 &= c_2a + e_2 \\ x_3 &= c_3a + k_3b + e_3 \\ x_4 &= c_4a + k_4b + e_4 \end{aligned}\right\} \qquad \text{[See 45]}$$

it is required to prove that the following is a synonymous system

$$\left.\begin{aligned} x_1 &= C_1c + K_1d + e_1 \\ x_2 &= C_2c + K_2d + e_2 \\ x_3 &= C_3c + e_3 \\ x_4 &= C_4c + e_4 \end{aligned}\right\} \qquad [48]$$

If the system [45] holds, then

$$\left.\begin{aligned} r_{12} &= \alpha_1\,\alpha_2 \\ r_{13} &= \alpha_1\,\alpha_3 \\ r_{14} &= \alpha_1\,\alpha_4 \\ r_{23} &= \alpha_2\,\alpha_3 \\ r_{24} &= \alpha_2\,\alpha_4 \\ r_{34} &= \alpha_3\,\alpha_4 + \beta_3\,\beta_4 \end{aligned}\right\} \dots\dots\dots\dots [49]$$

If [48] holds, then

$$\left.\begin{aligned} r_{12} &= \gamma_1\,\gamma_2 + \delta_1\,\delta_2 \\ r_{13} &= \gamma_1\,\gamma_3 \\ r_{14} &= \gamma_1\,\gamma_4 \\ r_{23} &= \gamma_2\,\gamma_3 \\ r_{24} &= \gamma_2\,\gamma_4 \\ r_{34} &= \gamma_3\,\gamma_4 \end{aligned}\right\} \dots\dots\dots\dots [50]$$

in which the γ's and the δ's are defined in a manner similar to that for the α's and β's. If the following substitutions (discovered by the writer after a mathematical development which was undoubtedly needlessly indirect and therefore not here repeated) for the factors in system [49] are made,

$$\alpha_1 = \gamma_1\,g$$

$$\alpha_2 = \gamma_2\,g$$

$$\alpha_3 = \gamma_3\,g$$

$$\alpha_4 = \gamma_4\,g$$

$$g^2 = \frac{\gamma_1\,\gamma_2 + \delta_1\,\delta_2}{\gamma_1\,\gamma_2}$$

$$\beta_3\,\beta_4 = \frac{\delta_1\,\delta_2\,\gamma_3\,\gamma_4}{\gamma_1\,\gamma_2 + \delta_1\,\delta_2}$$

then the system [49] will be found to transform into system [50] thus proving the proposition.

It is thus futile to look to the six inter-correlations for evidence as to whether x_1 and x_2, or x_3 and x_4 possess the second general factor.

We may obtain a suggestion upon this point by study of the coefficients of correlation corrected for attenuation between x_1 and x_2 and between x_3 and x_4. If system [49] holds and if x_∞, x_ω, x_τ, x_υ stand for the true scores in x_1, x_2, x_3, x_4, respectively, then it is probable that $r_{\infty\omega} > r_{\tau\upsilon}$, while if the system [50] holds, in all probability $r_{\tau\upsilon} > r_{\infty\omega}$, while if they are about equal and each high the inter-correlations may readily have arisen from system [51] following, in which a, b, c, and the e's are independent.

$$\left.\begin{aligned} x_1 &= c_1 a + j_1 c + e_1 \\ x_2 &= c_2 a + j_2 c + e_2 \\ x_3 &= c_3 a + k_3 b + e_3 \\ x_4 &= c_4 a + k_4 b + e_4 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots [51]$$

This system may, by appropriate substitutions, be transformed into either [45] or [48]. It leads to the same objective situation, namely, $t_{1\,2\,3\,4} = t_{1\,2\,4\,3} \neq 0$ and $t_{1\,3\,4\,2} = 0$ as do systems [45] and [48]. Adopting the law of parsimony we will not resort to this system [51] if systems [49] and [50] suffice.

When the situation $t_{1\,2\,3\,4} = t_{1\,2\,4\,3} \neq 0$ arises we may utilize other variables and calculate tetrad differences involving x_1, x_2, x_i, and x_j, and again involving x_3, x_4, x_i, and x_j, with the expectation that if the special factor b is found in x_1 and x_2 and not in x_3 and x_4 the tetrad difference $t_{1\,2\,i\,j} = t_{1\,2\,j\,i} \neq 0$ whereas $t_{3\,4\,i\,j} = t_{3\,4\,j\,i} = 0$. If this is done for all combinations of the variables four at a time, in which a second factor b has been found to run through two only of four variables we may obtain information as to which variables have special bonds as illustrated in Table VI. In this table the two numbers in the captions of the columns indicate two of the variables involved in a tetrad difference as given in Table V. The entries in the columns give, to the first two decimal places, the median absolute value of the three tetrad differences arising from a certain four variables. For example in Table V the median tetrad difference for variables x_1, x_2, x_3, x_4 is .25, and since the smallest tetrad difference is $t_{1\,3\,4\,2}$ this indicates a special bond between x_1 and x_2 or between x_3 and x_4. Accordingly the .25 is written in Table VI once in column 12 and a second time in column 34. The same manner of entry has been followed for all the other median tetrad differences of Table VI.

The larger the sums of the columns the greater the indication of a special bond between the variables indicated in the captions. This table will aid us in the future study of special factors.

If from a study of the four variables x_1, x_2, x_3, x_4 we conclude that a special bond exists between either x_1 and x_2 or between x_3 and x_4 we may secure presumptive evidence as to which of these is the case if a single additional variable is available, x_5, by substituting it in turn for x_1, x_2, x_3, x_4. If the bond is between x_1 and

TABLE VI

Median Tetrad Differences Allocated to the Pairs of Variables From Which They May Have Arisen

12	13	14	15	16	17	18	19	23	24	25	26
25	13	7			6					13	11
8	15	8								6	
17	6									8	
14	10									5	
17	15										
22	12										
38	12										
33											
23											
22											
31											
26											
21											
36											
26											
29											
388	83	15			6					32	11

TABLE VI—*Continued*

27	28	29	34	35	36	37	38	39	45	46	47
14	10	12	25		13		7	8	8	17	14
		6	13		6				8		
			11		8						
			14		12						
			10		11						
			12		10						
			27								
			27								
			19								
			17								
			21								
			15								
			28								
			22								
			21								
14	10	18	282		60		7	8	16	17	14

48	49	56	57	58	59	67	68	69	78	79	89
17	22	38	33	23	22	31	26	21	36	26	29
6	6	13	15	6	10	21	15		15	12	12
7	8	6	27	19	17	8			5	22	21
	14	27	13	6	8				28	11	10
	10	16	7						12	10	17
		10	14						10		
		17							16		
30	60	127	109	54	57	60	41	21	122	81	89

x_2, system [52] below holds, and if between x_3 and x_4, system [53] holds.

If the b factor lies in x_1 and x_2 then

$$\left.\begin{aligned} t_{1234} &= t_{1243} \neq 0 \\ t_{1235} &= t_{1253} \neq 0 \\ t_{1245} &= t_{1254} \neq 0 \\ t_{1534} &= t_{1543} = 0 \\ t_{2534} &= t_{2543} = 0 \end{aligned}\right\} \qquad [52]$$

If the b factor lies in x_3 and x_4 then

$$\left.\begin{aligned} t_{1234} &= t_{1243} \neq 0 \\ t_{1235} &= t_{1253} = 0 \\ t_{1245} &= t_{1254} = 0 \\ t_{1534} &= t_{1543} \neq 0 \\ t_{2534} &= t_{2543} \neq 0 \end{aligned}\right\} \quad \text{[53]}$$

It was stated that the evidence was presumptive only. The reason for this qualification lies in the fact that when x_5 is substituted for x_4 it is not necessarily true that the a factor running through the four variables x_1, x_2, x_3, x_4 is the same as the a factor running through x_1, x_2, x_3, x_5. If, however, when not only x_5 is substituted, but also when x_6, x_7, etc., are substituted, systems such as [52] and [53] always indicate that x_1 and x_2 possess a special bond and not x_3 and x_4, the evidence is practically conclusive that the special bond lies in x_1 and x_2 and not in x_3 and x_4. We may illustrate this by a study of variables x_1, x_2, x_6, x_9. Variables x_6 and x_9 have here been chosen because the data of Table VI suggest that these latter are not possessed of a special bond. We have

$$t_{1269} = .21 \qquad t_{1296} = .22 \qquad t_{1692} = .01$$

This indicates a special bond between x_1 and x_2 or between x_6 and x_9. We will first use x_3 for the fifth variable. We then have

$t_{1236} = .07$	$t_{1263} = .16$	$t_{1692} = .08$	(a)
$t_{1239} = -.05$	$t_{1293} = .03$	$t_{1392} = .08$	(b)
$t_{1369} = .10$	$t_{1396} = .06$	$t_{1693} = -.04$	(c)
$t_{2369} = -.03$	$t_{2396} = -.09$	$t_{2639} = -.06$	(d)

The row (a) throws no light upon the issue. It rather suggests that some special bond exists between x_3 and one of the variables

x_1, x_2, and x_6. The same inconclusive evidence is present in rows (*b*), (*c*), and (*d*), for the probable error of these tetrad differences is in the neighborhood of .035. We will repeat the process using x_4 in place of x_3. We have

$t_{1246} = .17$	$t_{1264} = .17$	$t_{1462} = .00$	(*a*)
$t_{1249} = .22$	$t_{1294} = .22$	$t_{1492} = .00$	(*b*)
$t_{1469} = -.06$	$t_{1496} = -.03$	$t_{1692} = .03$	(*c*)
$t_{2469} = -.08$	$t_{2496} = -.05$	$t_{2694} = .03$	(*d*)

Rows (*a*) and (*b*) very clearly indicate that the special bond exists between x_1 and x_2, while rows (*c*) and (*d*) give tetrads deviating from 0 by amounts quite readily attributable to chance and thus scarcely indicate a bond between x_6 and x_9. Let us, however, continue further using x_5 as our fifth variable. The tetrads are

$t_{1256} = .40$	$t_{1265} = .38$	$t_{1562} = -.01$	(*a*)
$t_{1259} = .22$	$t_{1295} = .22$	$t_{1592} = -.01$	(*b*)
$t_{1569} = -.02$	$t_{1596} = -.06$	$t_{1695} = -.04$	(*c*)
$t_{2569} = .00$	$t_{2596} = -.06$	$t_{2695} = -.07$	(*d*)

Here again the evidence is very strong that the special bond is between x_1 and x_2 and not between x_6 and x_9. Continuing similarly with x_7

$t_{1267} = .32$	$t_{1276} = .31$	$t_{1672} = -.01$	(*a*)
$t_{1279} = .26$	$t_{1297} = .28$	$t_{1792} = .02$	(*b*)
$t_{1769} = .00$	$t_{1796} = .01$	$t_{1697} = .00$	(*c*)
$t_{2769} = -.02$	$t_{2796} = -.03$	$t_{2697} = -.01$	(*d*)

Again the special bond is indicated as lying between x_1 and x_2 and not between x_6 and x_9. Finally we will use x_8 as the odd variable

$$t_{1268} = .25 \qquad t_{1286} = .26 \qquad t_{1682} = .00 \quad (a)$$
$$t_{1289} = .29 \qquad t_{1298} = .30 \qquad t_{1892} = .01 \quad (b)$$
$$t_{1869} = -.04 \qquad t_{1896} = -.01 \qquad t_{1698} = .03 \quad (c)$$
$$t_{2869} = -.05 \qquad t_{2896} = -.03 \qquad t_{2698} = .02 \quad (d)$$

Here again the special bond is indicated as lying between x_1 and x_2.

This concludes the number of such tests of the question. We are clearly justified in saying that a special bond exists between x_1 and x_2. If we study Tests 1 and 2 we are warranted in concluding that this bond is of the nature of "facility with verbal material." Further, since no special bond between x_6 and x_9 has as yet been indicated these particular variables will prove valuable to use in the search for other special bonds.

Finally if it is true that there is no special bond between x_6 and x_9, and if the special b factor in x_1 and x_2 is the only special b factor entering into either of these variables, then every pentad function p_{1269j} where x_j may be x_3, x_4, x_5, x_7, or x_8 will equal 0. The various pentads are:

$$p_{12369} = .0034$$
$$p_{12469} = \text{small}$$
(It is known to be small because one set of three tetrads involving four of these five variables yields three small values.)
$$p_{12569} = .0006$$
$$p_{12679} = \text{small}$$
$$p_{12689} = \text{small}$$

The pentad function p_{12369} is probably about one and one-half times its standard error, so it is likely there is a small added special factor between x_1 and x_3 or between x_2 and x_3. Reference to Table VI indicates that it is likely that this small bond is between x_1 and x_3 and not between x_2 and x_3. If this is the case the bond is presumably of the nature of "speed."

This chapter is concerned with the presentation of methods rather than the detailed investigation of the data, so we will not here repeat steps, such as those just concluded, in order to find if there exists a special bond between x_1 and x_3.

Let us, however, attempt to ascertain the relative sizes of the general factor a and of the special factor b involved in x_1 and x_2. We have established that for x_1, x_2, x_6, x_9 we may with high probability write down

$$x_1 = c_1 a + k_1 b + e_1$$

$$x_2 = c_2 a + k_2 b + e_2$$

$$x_6 = c_6 a + e_6$$

$$x_9 = c_9 a + e_9$$

giving

$$r_{12} = \alpha_1 \alpha_2 + \beta_1 \beta_2 \quad \text{(i)} \qquad r_{26} = \alpha_2 \alpha_6 \quad \text{(iv)}$$

$$r_{16} = \alpha_1 \alpha_6 \quad \text{(ii)} \qquad r_{29} = \alpha_2 \alpha_9 \quad \text{(v)}$$

$$r_{19} = \alpha_1 \alpha_9 \quad \text{(iii)} \qquad r_{69} = \alpha_6 \alpha_9 \quad \text{(vi)}$$

From (ii) and (iii) we obtain

$$\frac{\alpha_6}{\alpha_9} = \frac{r_{16}}{r_{19}} = \frac{.2318}{.1960} = 1.18$$

From (iv) and (v) we obtain . . . average = 1.13

$$\frac{\alpha_6}{\alpha_9} = \frac{r_{26}}{r_{29}} = \frac{.2988}{.2770} = 1.08$$

From (vi) we have

$$r_{69} = \alpha_6 \alpha_9 = (1.13\alpha_9)(\alpha_9)$$

yielding

$$\alpha^2_9 = \frac{r_{69}}{1.13} = \frac{4351}{1.13} = .385$$

Thus

$$\alpha_9 = \pm .62 \text{ and } \alpha_6 = \pm .70$$

Since α_6 and α_9 must have the same sign because r_{69} is greater than 0 and since it does not matter which sign we attach to these α's we will choose the positive sign. From (v)

From (iv)

$$\left.\begin{aligned} \alpha_2 &= \frac{r_{29}}{\alpha_9} = \frac{.2770}{.62} = .447 \\ \alpha_2 &= \frac{r_{26}}{\alpha_6} = \frac{.2988}{.70} = .427 \end{aligned}\right\} \ldots \text{ average} = .437$$

From (iii)

From (ii)

$$\left.\begin{aligned} \alpha_1 &= \frac{r_{19}}{\alpha_9} = \frac{.1960}{.62} = .316 \\ \alpha_1 &= \frac{r_{16}}{\alpha_6} = \frac{.2318}{.70} = .331 \end{aligned}\right\} \ldots \text{ average} = .324$$

From (i)

$$\beta_1 \beta_2 = r_{12} - \alpha_1 \alpha_2 = .6328 - .1415 = .4913$$

Thus if we assume temporarily that $\beta_1 = \beta_2$, each is equal to $\sqrt{.4913}$, or .70. It is noteworthy that the special *b* factor in these two tests is much larger than the general *a* factor. It should again be emphasized that this calculation leads merely to presumptive values of α_1, α_2, α_6, α_9, β_1, β_2, though considerable support to the procedure is given by the fact that the three-times averages have been taken (yielding 1.13, .437, and .324) and the values averaged have differed by such small amounts that the differences could well be attributed to chance.

Having ascertained that a special factor exists between x_1 and x_2 we may investigate the remaining variables for other *b* factors. Table VI immediately suggests that x_3 and x_4 should be studied. The tetrad differences which could be influenced by the $x_1\, x_2$ bond will be omitted from such a study. A study of this nature does indicate an $x_3 x_4$ bond. If x_1, x_2, x_3, x_4 may be represented by such a system as [51], page 71, there is no point in investigating these four variables by [35], for as already pointed out it would be known that they could be expressed as indicated in system [45] or system [48], page 69. It would be profitable to express x_1, x_2, x_3, x_4 as simply due to an *a* and a *b* bond, were such an explanation also serviceable in other connections. To determine if this is the case, we must add a fifth variable, x_j, determine the pentad function p_{1234j}, and ascertain if in general two bonds are sufficient. Table X yields the following pentad functions.

$$p_{12345} = -.0028$$

$$p_{12346} = .0004$$

$$p_{12347} = -.0056$$

$$p_{12348} = -.0003$$

$$p_{12349} = -.0073$$

$$\sigma_{p_{12347}} = .0025$$

Presumably two of these pentads are less than their probable errors (though of this we cannot be absolutely certain without calculation, for $\sigma_{p_{12347}}$ alone has been calculated), while the other three are apparently considerably larger than their probable errors. On the whole the results indicate that when x_1, x_2, x_3, x_4 are combined with other variables the total situation demands for its explanation not less than three general factors.

The type of analysis thus far illustrated in this chapter may be pursued farther to secure suggestions of still additional bonds. It does, however, fail as a desired technique when the number of factors exceeds three, and it even is insufficient for two or three factors in that it does not tell just where the various factors are located and what are the numerical values of their standard deviations. On these accounts another method has been developed. It will be illustrated by an example using the basic data for the population of 140 seventh-grade children given in the next chapter.

The steps may be outlined as follows: (*a*) Determine tentative standard deviation values for the various factors in each variable. This may be done as in chapters iv, v, etc., by finding the mean value of all the tetrads involving $x_1 x_2$, again of those involving $x_1 x_3$, etc., and tentatively assigning factor values so that the bonds, other than that given by a general factor, running throughout all the variables, are largest for those pairs of variables giving the largest mean tetrads. Further, this may be done by utilizing the propositions already given covering three, four, and five variables. (*b*) Refine these preliminary values so as to reduce the mean square error of estimate of the inter-correlation coefficients.

We will start with the preliminary factor values as given in Table XI, chapter iv, and here describe the steps involved in their refinement.

The calculation employs the method of least squares and requires for its serviceability that corrections to these preliminary factor values be small with reference to the values themselves, for the second and higher power terms are neglected in comparison with first power terms. Further, since there are only slightly over thirty-six equations of condition, represented by the thirty-six inter-correlations and such reliability coefficients as can be employed, it is obvious that the number of factor values assigned should be less than this for the least square determination must have fewer independent variables than equations of condition. The cells starred in Table XI total twenty-six and adding those double starred (as explained shortly) we have a total of thirty-one, so that we are quite clearly approaching the upper limit of the number of independent factors which it is worth investigating. The problem now is how closely will twenty-six (or thirty-one) independent factors located in the cells noted describe the complex situation represented by the inter-correlations and reliability coefficients which have been found. The solution obtained after a number of successive approximations yields the values given in Table XII. We will now explain the method of reaching this least square solution. The preliminary estimate of the size of α_1 is .3. Let us for the moment assume that all the other factors are correctly estimated and that this one is wrong, and that it should be $(.3 + a)$ in which a^2 is negligible in comparison with a. Then we have

$$r_{1I} = (.3 + a)^2 + (.7)^2 + (.2)^2 + (\sigma^2_{E_1})$$

from which

$$2(.3)a = (r_{1I} - \sigma^2_{E_1}) - .09 - .49 - .04$$

Assuming as explained in chapter iv, page 102, that $\sigma^2_{E_1}$ is at least equal to $.05 r_{1I}$ we have

$$\begin{aligned} 2(.3)a &\leqq (.95 r_{1I} - .62) \\ &\leqq [.95\ (.92) - .62] \\ &\leqq .25 \end{aligned}$$

As long as a is such a value that

$$2(.3)a < .25$$

we will neglect this condition, but if $2(.3)a > .25$ then the first of our condition equations will be $2(.3)a = .25$. In other words the difference from zero of the following expression

$$[.25 - 2(.3)a] \ldots\ldots$$

is the residual or error of estimate. For the second condition equation we have

$$r_{12} = (.3 + a)\ (.4) + (.7)\ (.7)$$

yielding, since $r_{12} = .63$

$$[.02 - (.4)a]$$

as the residual or error of estimate. A similar condition equation may be written for every other correlation coefficient involving x_1, giving eight condition equations coming from inter-correlation coefficients and one which is occasionally of service coming from a reliability coefficient.

We now wish to so determine a that the sum of the squares of the errors of estimate shall be a minimum. This is readily done if we will first express the condition equations in symbolic terms. Let α_1 be the preliminary estimate of the general factor in x_1; α_2 of the general factor in x_2; . . . . α_9 of the general factor in x_9; β_2 of the second factor in x_2; etc.; and let e_{12} be the residual error in the condition equation involving r_{12}; e_{13} the error in the condition equation involving r_{13}; etc.; e_{1I} the error in the condition equation involving r_{1I} (when r_{1I} is used). Then the eight (or nine) condition equations are

$$\begin{aligned} e_{12} = r_{12} &- \alpha_1 \alpha_2 - \beta_1 \beta_2 - \gamma_1 \gamma_2 \\ &- \delta_1 \delta_2 - \varepsilon_1 \varepsilon_2 - \xi_1 \xi_2 \\ &- \alpha_2 a_1 \end{aligned}$$

which, if we let ϱ_{12} stand for the residual in the case of the preliminary estimation, we may write

$$\left.\begin{aligned} & e_{12} = \varrho_{12} - \alpha_2 a_1 \\ \text{Similarly} \quad & e_{13} = \varrho_{13} - \alpha_3 a_1 \\ & \quad \cdot \quad \cdot \quad \cdot \\ & \quad \cdot \quad \cdot \quad \cdot \\ & \quad \cdot \quad \cdot \quad \cdot \\ & e_{19} = \varrho_{19} - \alpha_9 a_1 \end{aligned}\right\} \qquad [54]$$

$$e_{1I} = \varrho_{1I} - 2\alpha_1 a_1 \qquad [55]$$

Let us square these residuals and sum, in order to determine the value of a yielding the least-square error. We will consider two cases, first when [55] is not involved, and second when it is. For the first case we have

$$\Sigma\, e^2_{1p} = \Sigma(\varrho_{1p} - \alpha_p a_1)^2 \qquad [56]$$

in which the subscript p takes all values from one to nine except the value 1. Differentiating this function with respect to a_1 and setting the derivative equal to 0 will give us the required value of a_1.

$$\frac{d\Sigma e^2_{1p}}{da_1} = \Sigma\, 2(\varrho_{1p} - \alpha_p a_1)\ (-\alpha_p) = -2(\Sigma\alpha_p\, \varrho_{1p} - a_1\Sigma\alpha^2_p) = 0$$

from which

$$a_1 = \frac{\Sigma\alpha_p\, \varrho_{1p}}{\Sigma\alpha^2_p}$$

If we let $\Sigma\alpha^2$ equal Σ of all the α^2 values, including α^2_1, this equation may be written in the following form which is more simple to use

$$a_1 = \frac{\Sigma\alpha_p\, \varrho_{1p}}{(\Sigma\alpha^2) - \alpha^2_1} \quad \left.\begin{matrix}\text{(to be used when } r_{1I} \\ \text{is not involved)}\end{matrix}\right\} \ldots\ldots\ldots\ldots [57]$$

For the second case we readily obtain

$$a_1 = \frac{(\Sigma\alpha_p\, \varrho_{1p}) + 2\varrho_{1I}\, \alpha_1}{(\Sigma\alpha^2) + 3\alpha^2_1} \quad \left.\begin{matrix}\text{(to be used when } r_{1I} \\ \text{is involved)}\end{matrix}\right\} \ldots\ldots\ldots [58]$$

Equation 57 or 58 gives the correction in the preliminary estimate which is required, provided the values of all other factors are correct.

Of course, the values of these other factors are not correct, so the best way to proceed is to calculate $a_1, a_2, \ldots a_9, b_1, b_2, \ldots b_9, \ldots f_1, f_2, \ldots f_9$, using throughout the preliminary weights and correct for the one of these which is found to be the largest. Having done this, recalculate and correct for the one which is then the largest, and continue until the corrections become negligible. This is the procedure here followed except that at times several corrections were made before the next recalculation. The making of several corrections has led to a rather rapid convergence toward the ultimate values, but the writer hesitates to propose rules which may be universally trusted in determining whether convergence takes place when several corrections are made at a single time. The values of the factors thus finally obtained are given in Table XII, chapter iv. From the standpoint of a strictly accurate least-square solution the writer believes that these are correct to two decimal places. This does not mean that the probable error is in the third decimal place. The value of the probable error is not known, but it may well be in the neighborhood of .08. It is probably larger for the small factor values than for the larger ones.

The mathematical functions here dealt with lead directly into the field of n-dimensional trigonometry, and are undoubtedly amenable to more extensive and refined treatment than has here been given to them. The writer is unable to say that the specific 31 cells of Table XI, page 103, which have entries in them, are the only 31 which could be chosen and yield as excellent a fit as is shown by Table XIII. He believes this to be the case because (*a*) starting out with other initial cell values and following the procedure of correction already described there seems to be a convergence toward these single final values; also, (*b*) choosing cells either singly or in pairs by chance and values therefore by some arbitrary rule (for example, every value entered or taken out of a cell shall be of magnitude .3 for the first 100 entries, of magnitude .2 for the next 100, of magnitude .1 for the next 100, of magnitude .05 for the next 100, etc., and entering or taking out values depending upon whether the process decreases or increases the sum of the square errors) seems to lead to the same net result. The expression "seems to lead" has been used instead of "does lead," because the arithmetical labor of this process (*b*) is so great that the writer has not carried such a calculation to a determinate conclusion. The tendency, however, seems clear, and he hopes later, either by empirical or more conclusive analytical means to show that when the number of cells to be filled is fixed there is a unique solution to the problem, which are the cells and what are the values within them in order to obtain a minimal square error of estimate.

We have now noted all of the basic mathematical techniques which we will need to employ except the calculation of the mean of the tetrads involving a given pair of variables and the calculation of the standard error of such a mean tetrad. The calculation of the mean tetrad itself is very simple and is illustrated in subsequent chapters, but the calculation of a standard error of a mean tetrad is sufficiently involved to make it appropriate subject-matter for this chapter dealing with statistical techniques.

Were one to choose one at random from all possible tetrads coming from Table X, its standard error would be given by Formula 28, page 49. Let us consider the tetrad differences explicable by the bond $x_1 x_2$. There are 30 such, and calculation shows that 22 are positive and 8 are negative. If we take any one of these thirty at random, its probable error is given by Formula 28, providing our choice of this $x_1 x_2$ bond is a random choice. Should each of thirty people choose a tetrad by chance and in such a manner that all tetrads were finally chosen, and should each draw a conclusion as to the significance of the tetrad difference found, widely different inferences as to the reality of an $x_1 x_2$ bond would be made, but the median of these thirty conclusions would be the conclusion drawn by the person who had investigated the median tetrad difference. His conclusion is the most trustworthy one if a conclusion is to be drawn from a single tetrad. However, using but a single tetrad uses but a part of the data. We have $t_{1\,2\,3\,4} = r_{12}\,r_{34} - r_{13}\,r_{24}$. While studying the $x_1 x_2$ bond, the first two subscripts of the tetrad, namely, 1 and 2, are fixed, but the remaining subscripts can take all possible permutations of the subscripts of the remaining variables.

The strength of the $x_1 x_2$ bond should be judged by this total situation which involves $(n-2)(n-3)$ tetrads, where n is the total number of variables. The obvious way to proceed is to calculate the mean of these tetrads and its standard error and thus determine the probability of its divergence from zero being significant. This will be done, and as a still briefer method, the median of these tetrads will be found and its standard error determined. Such a procedure should give us excellent evidence as to the presence of a factor other than a general factor in $x_1 x_2$, but it will not yield a measure of this factor, nor will it tell us whether or not there are several such additional factors. In short this procedure should be antecedent to the iteration process already detailed, pages 80–83.

The line of argument is straightforward, but there is a very real difficulty in following it out, due to the algebraic complexity and length of the standard error formula for the mean of the $(n-2)(n-3)$ tetrads. Since every tetrad is correlated with every other, the ordinary formula for the mean of $(n-2)(n-3)$ correlated measures is an expression having $[(n-2)(n-3)]^2$ terms, each term being a polynomial of four terms. All expanded terms except the square terms involve correlations between correlation coefficients which are in themselves lengthy algebraic expressions. It is thus clearly out of the question to calculate such a standard error without resorting to approximations. One would expect, however, to obtain fairly good results if terms are classified by type and approximations obtained for each type, providing the basic table of correlations is fairly homogeneous, for example, composed of positive coefficients all lying substantially between .0 and .7, as do the correlations of Table X. A standard error formula applicable under these conditions is derived herewith. We will determine the standard error of the mean of the tetrads involving the $x_1\,x_2$ bond. Let n equal the number of variables. Let subscripts i, j, k, h identify variables other than x_1 and x_2. There will be no repeated subscripts. Thus Σr_{ij} does not include a term r_{ii} or r_{jj}. Since x_2 enters into the situation in exactly the same manner as x_1, we will, when designating the averages, use a symbol with the subscript 1 to stand for the average of terms, having a subscript 1 or the subscript 2. Letting M, M', etc., stand for the number of things of the type in question, we will employ the following averages:

$$r_1 = \frac{1}{2M}\Sigma(r_{1i} + r_{2i}) \qquad [59]$$

$$r = \frac{1}{M'}(\Sigma\, r_{ij}) \qquad [60]$$

Let $\bar{r}_{ij}$ equal the true r_{ij} value, and let $\delta_{ij} = r_{ij} - \bar{r}_{ij}$. Let the δ's with other subscripts be similarly defined. We may designate product-moments as follows:

$$p_{ijkh} = \frac{1}{M''}\Sigma(\delta_{ij}\,\delta_{kh}) \qquad [61]$$

and similarly for the other product-moments. In the case of repeated subscripts we have

$$p_{ijij} = \sigma^2_{r_{ij}}$$

In this case we will not use the p notation but the more condensed σ^2 notation. As the variances of the various correlation coefficients will not differ greatly, we will use the average in place of the separate values. We have two types of product-moments: (a) one subscript repeated; (b) no subscript repeated. The average of all of the first type we will designate by the letter q, and the average of those of the second type by the letter p, and we will use these average values throughout in place of the separate values. Thus the symbols that will enter into the standard error formula are:

n, the number of variables

r_{12}, the correlation between x_1 and x_2

r_1, the mean correlation between x_1 and one other variable not x_2, or that between x_2 and one other variable not x_1

r, the mean correlation between two variables neither of which is x_1 or x_2

σ^2, the mean of the variances of the correlation coefficients

q, the mean product-moment of the errors in two correlation coefficients which involve three variables only

p, the mean product-moment of the errors in two correlation coefficients which involve four variables

Finally, there will enter N (do not confuse with n), the size of the population.

Let $t_{1\,2}$ = average tetrad, then we have

$$t_{1\,2} = \frac{1}{(n-2)\,(n-3)}\,(t_{1\,2\,3\,4} + t_{1\,2\,4\,3} + t_{1\,2\,3\,5} + \ldots + t_{1\,2\,3\,n} + t_{1\,2\,n\,3} + t_{1\,2\,4\,5} + t_{1\,2\,5\,4} + \ldots + t_{1\,2\,n\,4} + t_{1\,2\,5\,6} + t_{1\,2\,6\,5} + \ldots\ldots) \quad \ldots\ldots [62]$$

To generalize this, let us designate subscripts 3, 4, 5, 6, . . . by i, if only one such is involved, by i and j if two such are involved, and by i, j, k, and h if four such are involved. Let Δ_{12} represent an error in $t_{1\,2}$. Let Δ_{1234} represent an error in $t_{1\,2\,3\,4}$, etc. With this notation we have

$$\Delta_{12} = \frac{1}{(n-2)\,(n-3)}\,(\Delta_{12ij} + \Delta_{12ji} + \Delta_{12ik} + \Delta_{12ki} + \ldots + \Delta_{12kh} + \Delta_{12hk} + \ldots)$$

Squaring both sides, and summing and dividing by the population M'' will yield the variance desired.

$$(n-2)^2\,(n-3)^2\,\sigma^2_{t_{12}}$$

$$= \frac{1}{M''}\,\Sigma\Delta^2_{12ij} \ldots\ldots [(n-2)(n-3) \text{ summations each of this sort}]$$

$$+ \frac{1}{M''}\,\Sigma\Delta_{12ij}\,\Delta_{12ji} \ldots [(n-2)(n-3) \text{ summations each of this sort}]$$

$$+ \frac{1}{M''}\,\Sigma\Delta_{12ij}\,\Delta_{12ik} \ldots [(n-2)(n-3)(n-4)(n-5) \text{ summations each of this sort}]$$

$$+ \frac{1}{M''}\,\Sigma\Delta_{12ij}\,\Delta_{12kh} \ldots [4(n-2)(n-3)(n-4) \text{ summations each of this sort}] \ldots\ldots [63]$$

For brevity let us designate the four different types entering into the right-hand member of the equation by A, B, C, D. We must expand and sum each of these.

$$A = \frac{1}{M''}\,\Sigma\Delta^2_{12ij} = \frac{1}{M''}\,\Sigma(r_{12}\,\delta_{ij} + r_{ij}\,\delta_{12} - r_{1i}\,\delta_{2j} - r_{2j}\,\delta_{1i})^2$$

$$A = \frac{1}{M''}\,[r^2_{12}\,\Sigma\delta^2_{ij} + r^2_{ij}\,\Sigma\delta^2_{12} + r^2_{1i}\,\Sigma\delta^2_{2j} + r^2_{2j}\,\Sigma\delta^2_{1i} + 2r_{12}\,r_{ij}\,\Sigma\delta_{12}\,\delta_{ij} - 2r_{12}\,r_{1i}\,\Sigma\delta_{2j}\,\delta_{ij} - 2r_{12}\,r_{2j}\,\Sigma\delta_{1i}\,\delta_{ij} - 2r_{1i}\,r_{ij}\,\Sigma\delta_{12}\,\delta_{2j} - 2r_{2j}\,r_{ij}\,\Sigma\delta_{12}\,\delta_{1i} + 2r_{1i}\,r_{2j}\,\Sigma\delta_{1i}\,\delta_{2j}]$$

$$A = r^2_{12}\,\sigma^2 + r^2\,\sigma^2 + 2r^2_1\,\sigma^2 + 2r_{12}\,r\,p - 2r_{12}\,r_1\,q - 2r_{12}\,r_1\,q$$
$$- 2r_1\,rq - 2r_1\,rq + 2r^2_1\,p$$

$$A = r^2_{12}\,\sigma^2 + 2r^2_1\,\sigma^2 + r^2\sigma^2 - 4r_{12}\,r_1\,q + 2r_{12}\,r\,p + 2r^2_1\,p - 4r_1\,r\,q$$

We similarly find

$$B = r^2_{12}\,\sigma^2 + r^2\,\sigma^2 - 4r_{12}\,r_1\,q + 2r_{12}\,r\,p + 4r^2_1\,q - 4r_1\,r\,q$$

and similarly

$$C = r^2_1\,\sigma^2 + r^2\,\sigma^2 - 2r_{12}\,r_1\,q + 2r_{12}\,r\,p + 2r^2_1\,p + r^2_1\,q$$
$$+ r^2_{12}\,q - 2r_{12}\,r_1\,p - 4r_1\,r\,q$$

and similarly

$$D = r^2\,\sigma^2 - 4r_1\,r\,q + 2r_{12}\,r\,p + 2r^2_1\,p + 2r^2_1\,q + r^2_{12}\,p - 4r_{12}\,r_1\,p$$

Summing these four expressions, taking each the appropriate number of times, i.e., to one A there is one B, and there are $(n-4)(n-5)$ C's and $4(n-4)$ D's, we obtain for the variance of the mean of all the $t_{1\,2\,i\,j}$ tetrads the value

$$\sigma^2_{t12} = \left\{\frac{1}{(n-2)\;(n-3)}\right\}[2r^2_{12}\,\sigma^2 + (n^2 - 9n + 22)\;r^2_1\sigma^2$$
$$+ (n^2 - 5n + 6)\;r^2\,\sigma^2 + (n^2 - 9n + 20)\;r^2_{12}\,q$$
$$+ (4n - 16)\;r^2_{12}\,p - (2n^2 - 18n + 48)\;r_{12}\,r_1\,q$$
$$- (2n^2 - 2n - 24)\;r_{12}\,r_1\,p + (2n^2 - 10n + 12)\;r_{12}\,r\,p$$
$$+ (n^2 - n - 8)\;r^2_1\,q + (2n^2 - 10n + 10)\;r^2_1\,p$$
$$- (4n^2 - 20n + 24)\;r_1\,rq] \qquad [64]$$

Let us write this equation in vertical array and supply numerical values for the coefficients given by the terms for different values of n, thus making it readily available for determining the standard error of the mean tetrad for situations having different numbers of variables.

TABLE VII

Coefficients of the Terms in the First Column Corresponding to the Number of Variables Indicated in the Captions of the Other Columns

	4	5	6	7	8	9	10	11	12	13
$r^2_{12}\sigma^2$	2	2	2	2	2	2	2	2	2	2
$r^2_1\sigma^2$	2	2	4	8	14	22	32	44	58	74
$r^2\sigma^2$	2	6	12	20	30	42	56	72	90	110
$r^2_{12}q$	0	0	2	6	12	20	30	42	56	72
$r^2_{12}p$	0	4	8	12	16	20	24	28	32	36
$r_{12}r_1 q$	—8	—8	—12	—20	—32	—48	—68	—92	—120	—152
$r_{12}r_1 p$	0	—16	—36	—60	—88	—120	—156	—196	—240	—288
$r_{12}r p$	4	12	24	40	60	84	112	144	180	220
$r^2_1 q$	4	12	22	34	48	64	82	102	124	148
$r^2_1 p$	2	10	22	38	58	82	110	142	178	218
$r_1 r q$	—8	—24	—48	—80	—120	—168	—224	—288	—360	—440
Value of the term $\frac{1}{(n-2)\ (n-3)}$	½	⅙	1/12	1/20	1/30	1/42	1/56	1/72	1/90	1/110

It is necessary to adopt some reasonable procedure to obtain the approximate values of the variables entering into this standard error formula. The values r_{12}, r_1, and r have already been specifically defined. The variance of a correlation coefficient is σ^2. We desire a single value for this which we can use in place of the large number of slightly different values which in reality enter into the formula. In the calculation of the standard error of the mean $t_{1\,2\,i\,j}$ tetrad, r_{12} enters in as often as r_{ij}, and the two together as often as r_{1i} or r_{2i}. Therefore, if when calculating σ^2 by the formula

$$\sigma^2 = \frac{(1 - R^2)^2}{N}$$

we use for R the following value

$$R = \frac{1}{4}(r_{12} + 2r_1 + r)$$

we should secure a very reasonable value for σ^2.

The product-moment, where three variables are involved, has been designated q (the same as $p_{12:13}$ of Formula 66). It may be calculated by the aid of Formula 66, page 93. There are, however, a very large number of different values of q, depending upon which three correlation coefficients are employed. Considering the tetrad $t_{1\,2\,i\,j}$ we can have sets of three variables as follows: $12i$, $12j$, $1ij$, $2ij$. Variables x_1 and x_2 play as large a part as do x_i and x_j. Let us therefore choose for the variables from which to determine q both x_1 and x_2 and x_i, this last to be chosen from $(n-2)$ possible variables, because its correlations with x_1 and x_2 approximate r_1. Thus, if $\frac{x_{1i} + x_{2i}}{2}$ is calculated for each of the $(n-2)$ variables, and if the variables are arranged in order upon the basis of these magnitudes, the median of them will presumably be most nearly equal to r_1. This will determine the particular variable of the $(n-2)$ which is called x_i for the purpose of calculating q. After determining the variables we still have three values of q, depending upon the order in which they are taken, as reference to Formula 66 shows. If q takes quite different values, depending upon which of the three orders (a) x_i, x_1, x_2, (b) x_1, x_i, x_2, (c) x_2, x_i, x_1 is taken, use for q the average of the three. If inspection informs one which of these three values of q will be the median, then that one may be taken in place of the mean of the three, thus shortening the calculation.

The product-moment p given by Formula 65 involves four variables, x_1 and x_2 playing as large a part as x_i and x_j.

Thus two of these four variables should be x_1 and x_2. The other two, x_i and x_j, should constitute a fair sampling of the $(n-2)$ variables. Let us therefore choose them on the following basis: As above, arrange the $(n-2)$ variables in the order of the magnitude of the quantities $\frac{r_{1i} + r_{2i}}{2}$. We will then choose the variables yielding the lower and upper quartile values, as x_i and x_j. If p takes quite different values, depending upon which of the three orders (*a*) x_1, x_2, x_i, x_j, or (*b*) x_1, x_i, x_2, x_j, or (*c*) x_1, x_j, x_2, x_i is taken, use for p the average of the three. As in the case of q, the median of these three values may be taken in place of the mean if inspection informs one which is the median.

The procedure described should enable one to tell whether an obtained x_1x_2 bond is to be accounted for by chance. In many cases the existence of a bond which is not due to chance is relatively so certain that we may investigate the median tetrad difference instead of the mean difference, and if the difference is so small that we need its probable error, we can multiply that given for the mean tetrad difference by 1.25 and have a good approximation to the probable error of the median tetrad difference. Though medians have been employed quite extensively in the succeeding chapters, they are not to be preferred to means. Following the procedure described yields a standard error of .018 for .019, the mean x_1x_2 tetrad difference, for the data of Table X.

Formulas for the calculation of certain product-moments.— There are many situations when the correlation between two correlation coefficients is needed. However, in general, it is not $r_{r_{12}\,r_{34}}$ which is needed but the product-moment $\sigma_{r_{12}}\,\sigma_{r_{34}}\,r_{r_{12}\,r_{34}}$. This product-moment is a function of the population as well as of the inter-correlations between the variables. Since

$$\sigma_r = \frac{1 - r^2}{\sqrt{N}}$$

we may take $1/N$ out as a factor of the entire product-moment and designate the balance by $p_{12:34}$, thus

$$\sigma_{r_{12}}\,\sigma_{r_{34}}\,r_{r_{12}\,r_{34}} = \frac{1}{N}(1 - r^2_{12})\,(1 - r^2_{34})\,r_{r_{12}\,r_{34}} = \frac{1}{N}p_{12:34}$$

If we expand this product-moment we have

$$\begin{aligned} p_{12:34} = {} & r_{13}\,r_{24} + r_{14}\,r_{23} \\ & - (r_{12}\,r_{13}\,r_{14} + r_{12}\,r_{23}\,r_{24} + r_{13}\,r_{23}\,r_{34} + r_{14}\,r_{24}\,r_{34}) \\ & + \tfrac{1}{2}r_{12}\,r_{34}\,(r^2_{12} + r^2_{14} + r^2_{23} + r^2_{34}) \ldots\ldots\ldots\ldots\ldots [65] \end{aligned}$$

The general product-moment is, as shown, a function of six correlation coefficients, and it is therefore something of a task to calculate the fifteen such that are required in the formula for the standard error of a pentad function. When this particular product-moment is needed [65] must be employed to get it.

In many cases a product-moment of the type $p_{12:13}$ is required. If we let

$$a = \frac{r_{12}\, r_{13}}{2}, \text{ and } b = 1 - r^2_{12} - r^2_{13}$$

this may be written

$$p_{12:13} = -ab + r_{23}\, b + r^2_{23}\, a \qquad [66]$$

This is a convenient equation to use in the calculation of $p_{12:13}$, which is the quantity called q in Formula 64. No less than thirty product-moments of this sort are required in the calculation of the standard error of the pentad function.

Method for determining the optimum scoring scheme.—In a test such as the arithmetic computation speed test, a number of possible scoring schemes seem more or less reasonable. Let X equal the number of attempts or responses, Y the number of errors or incorrect responses. Then the score might be X, the number of attempts, or $(X - Y)$, the number of correct responses, or $(X - 2Y)$, the number of correct responses minus the number of incorrect responses, or still some other combination. If the optimum combination of X and Y is linear we may set the score as equal to $(X + cY)$, where c is an undetermined constant, and then search for an appropriate condition to impose which will give us the best value of c. If we have a criterion measure, the value of c giving us the highest multiple correlation with the criterion will be the best value. If we do not have a criterion measure we may impose the condition that c shall take the value that gives the resulting score $(X + cY)$ the highest reliability. This is the condition that has here been used to determine the fitting combination of sub-scores, the number attempted, the number right, number wrong, number omitted, etc., to yield a single score for the entire test. If x and y stand for $(X - M_x)$ and $(Y - M_y)$, respectively, and if the subscripts 1 and 3 refer to the sub-scores on the first form, and subscripts 2 and 4 to the sub-scores on the second form of the test, then the correlation which is to be a maximum is that between $(x_1 + cy_3)$ and $(x_2 + cy_4)$. It is given by the following equation:

$$r_{(1+c3)\ (2+c4)} = \frac{\Sigma x_1 x_2 + c\Sigma x_1 y_4 + c\Sigma y_3 x_2 + c^2\Sigma y_3 y_4}{\sqrt{\Sigma x^2_1 + 2c\Sigma x_1 y_3 + c^2 \Sigma y^2_3}\ \sqrt{\Sigma x^2_2 + 2c\Sigma x_2 y_4 + c^2 \Sigma y^2_4}}$$

Using the usual notation in which

$$\sigma^2_1 = \frac{\Sigma x^2_1}{N},\ \sigma^2_3 = \frac{\Sigma y^2_3}{N},\ \text{etc., } p_{12} = \frac{\Sigma x_1 x_2}{N},\ p_{13} = \frac{\Sigma x_1 y_3}{N},\ \text{etc.,}$$

$$r_{(1+c3)\ (2+c4)} = \frac{p_{12} + c\ (p_{14} + p_{23}) + c^2 p_{34}}{\sqrt{\sigma^2_1 + 2cp_{13} + c^2\sigma^2_3}\ \sqrt{\sigma^2_2 + 2cp_{24} + c^2\sigma^2_4}}$$

If we take the derivative of this function and set it equal to zero, we will be able to determine the value of c which makes it a minimum. The required derivative is a quartic in c as follows:

$$\begin{aligned}
&\sigma^2_1\sigma^2_2\ (p_{14} + p_{23}) - p_{12}\ (\sigma^2_1 p_{24} + \sigma^2_2 p_{13}) \\
&\quad + [2\sigma^2_1\sigma^2_2 p_{34} + (\sigma^2_1 p_{24} + \sigma^2_2 p_{13})\ (p_{14} + p_{23}) \\
&\qquad - p_{12}\ (\sigma^2_1\sigma^2_4 + \sigma^2_2\sigma^2_3 + 4p_{13}p_{24})]\ c \\
&\quad + [3(\sigma^2_1 p_{24} + \sigma^2_2 p_{13})\ p_{34} - 3(\sigma^2_3 p_{24} + \sigma^2_4 p_{13})\ p_{12}]c^2 \\
&\quad + [(\sigma^2_1\sigma^2_4 + \sigma^2_2\sigma^2_3 + 4p_{13}p_{24})\ p_{34} \\
&\qquad - (\sigma^2_3 p_{24} + \sigma^2_4 p_{13})\ (p_{14} + p_{23}) - 2\sigma^2_3\sigma^2_4 p_{12}]\ c^3 \\
&\quad + [(\sigma^2_3 p_{24} + \sigma^2_4 p_{13})\ p_{34} - \sigma^2_3\sigma^2_4\ (p_{14} + p_{23})]c^4 = 0 \quad \ldots [67]
\end{aligned}$$

In almost all equations this quartic has been found to have two imaginary roots. In fact if we have similar forms, so that x_1 and x_2

are similar measures, and also x_3 and x_4, then r_{12} and r_{34} are each greater than zero, and they bear such a relationship to the other correlation coefficients that the quartic then has only two real roots. One of the real roots gives the value of c for which the reliability is a maximum, and the other for which it is a minimum.

Though Equation 67 was used a good many times in getting optimum combinations for the third and seventh grades separately, as reported in chapter viii, differences between these optimum reliabilities and those resulting from non-fractional weightings of the sub-scores were in all cases so small as not to warrant the use of the longer though more reliable scoring scheme. The technique for determining the optimum relationship between "rights" and "wrongs" (and at times also "omissions") gave throughout very reasonable results. Most of the scoring schemes adopted, because they contained integral weights and differed but slightly from the optimum schemes, are such as might readily have been hit upon by common sense, or whatever it is that guides one in the absence of data.

Under many conditions a somewhat simpler expression than [67] may be used to determine the optimum scoring scheme. If the two forms are genuinely similar, then, to a close approximation we may say that

$$r_{14} = r_{23}$$

$$r_{13} = r_{24}$$

$$\sigma^2_1 = \sigma^2_2$$

$$\sigma^2_3 = \sigma^2_4$$

Thus if we write

$$q^2 = \frac{\sigma^2_3}{\sigma^2_1} = \frac{\sigma^2_4}{\sigma^2_2}$$

$$\varrho_{14} = \frac{r_{14} + r_{23}}{2}$$

$$\varrho_{13} = \frac{r_{13} + r_{24}}{2}$$

Equation 67 becomes

$$2\varrho_{14} - 2\varrho_{13}\, r_{12} + qc\,[2r_{34} + 4\varrho_{13}\,\varrho_{14} - (2 + 4\varrho^2_{13})\, r_{12}]$$
$$+ 3q^2 c^2\,[2\varrho_{13}\, r_{34} - 2\varrho_{13}\, r_{12}] + q^3 c^3\,[(2 + 4\varrho^2_{13})\, r_{34}$$
$$- 4\varrho_{13}\,\varrho_{14} - 2r_{12}] + q^4 c^4\,[2\varrho_{13}\, r_{34} - 2\varrho_{14}] = 0 \ldots\ldots [68]$$

The accompanying tables of correlations and standard deviations used in determining the scoring scheme as given in chapter viii, page 153, may be utilized by one desiring practice in the use of this method.

TABLE VIII

MEANS, STANDARD DEVIATIONS, AND CORRELATIONS FOR DETERMINING THE OPTIMUM SCORING SCHEME FOR THE ARITHMETIC SPEED TEST FOR THE POPULATION OF 140 SEVENTH-GRADE PUPILS

	Form A Attempts	Form B Attempts	Form A Errors	Form B Errors
Means	69.96	79.79	.729	1.386
Standard Deviations	14.67	19.11	.925	2.371
Form B Attempts	.8229			
Form A Errors	.1495	.0916		
Form B Errors	.0661	.1901	.4191	

The equation to be solved according to [67] is

$$290{,}600 + 212{,}700\ c + 5{,}840\ c^2$$
$$+ 829.4\ c^3 + 1.591\ c^4 = 0$$

which gives

(attempts — 1.41 errors)

as the scoring scheme yielding the highest reliability.

CHAPTER IV

THE FACTORS FOUND IN THE SEVENTH-GRADE POPULATION OF 140

Many suggestions of independent mental traits are given in chapter i, and in chapter iii certain statistical tools for establishing the independence or similarity of different measures are described. This chapter reports an experimental investigation which utilizes the methods of chapter iii in an analysis of tests selected because of such suggestions of independence as provided in chapter i. The tests used are described in detail in chapter viii. When two or more tests are found to possess a common factor, a critical scrutiny of these tests and their method of administration may reveal its nature. Having secured an idea as to the nature of the independent mental factor, it is then in order to build up a test measuring this factor so far as possible uncontaminated by any other factors. It is to facilitate this step which is still to be done that the explanation of the tests is given in great detail in chapter viii. We will here refer to the tests only by their abbreviated titles and indicate why each was included in the investigation. The tests were given to the various groups as indicated by crosses in Table IX. The population of 140 consisted of 66 boys and 74 girls, all in the seventh grade. The mean age was 12.94 years and the standard deviation of ages was 1.048 years. There was a negligible difference in means and standard deviations for the boys and girls separately. The population of 109 seventh-grade pupils was a sub-population of the 140. The two third-grade populations were sub-samples of an original experimental population of 123 third-grade children of mean age 9.13 years and standard deviation 1.021 years. The number of boys was 70 and of girls 53. Though the sample of 60 is a select sample, the 110 may be considered a mere random sampling from the 123.

These particular tests were chosen expecting that: (*a*) if there is a verbal factor, as suggested by Thorndike and others, Tests 1 and 2 would betray it; (*b*) Tests 1, 3, and 10 should reveal a speed

TABLE IX

TABLE OF TESTS GIVEN TO EACH GROUP

Tests	140 Seventh-Grade Children	109 Seventh-Grade Children	110 Third-Grade Children	60 Third-Grade Children
Reading—				
1. Speed	×	×	×	×
2. Power	×	×	×	×
Arithmetic—				
3. Speed	×	×	×	×
4. Power	×	×	×	×
Memory—				
5. Words	×	×	×	×
6. Numbers	×	×	×	×
7. Meaningful symbols	×	×	×	×
8. Meaningless symbols	×	×	×	×
Space—				
9. Power	×	×	×	×
10. Speed		×		×
Interest—				
11. Intellectual		×		
12. Social		×		
13. Activity		×		

factor; (*c*) Tests 2, 4, 7, and 9 should reveal a general analytical capacity (abstract thought suggested by Thorndike); (*d*) Tests 3, 4, and 6 should reveal an arithmetic factor; (*e*) Tests 5, 6, 7, and 8 should reveal a memory trait; (*f*) Tests 7, 8, 9, and 10 should reveal a manipulation of spatial relationships trait; (*g*) Tests 11, 12, 13 should reveal traits less intellectual and more of an emotional nature. These tests have proved very serviceable for the purpose in hand, though a better battery could now be devised. One of the tests, No. 10, proved administratively weak, and as a consequence only a part of the pupils were tested by this measure. This and the fact that the interest tests were not given to all the pupils accounts for the reduced populations when Tests 10–13 are included. It is notable that some important traits suggested by the work reviewed are not represented in these tests. This was unavoidable because of expense, but a further study should certainly include tests measuring a number of additional traits.

The method followed in the analysis of the separate tests is, first, to secure an idea as to the location and strength of bonds, i.e., as to tests in pairs possessing some feature not commonly measured by the other tests; secondly, to write down preliminary estimates for each test, of the standard deviations of the separate factors measured by each; and, thirdly, to refine these preliminary estimates by an iteration process, securing final estimates which more closely (in the least-square error sense) describe the total set of given inter-correlations. For the population of 140 seventh-grade children the first step was accomplished by a technique which is both more involved and longer than that used on other populations. It is accordingly not given here and the reader is asked to take on faith the steps followed in securing the initial factor values for this seventh-grade population of 140 and to judge of the general possibility of securing such values by studying the procedure of the later chapters. We are, however, giving all the original data pertinent to the study.

The preliminary determination of the special bonds existing between the variables in pairs suggested that the order of magnitude was as follows: $x_1 x_2$; $x_3 x_4$; $x_5 x_6$ tied with $x_7 x_8$; $x_8 x_9$. Somewhere in this series, though not determined by the preliminary investigation, should be found $x_5 x_7$ and also $x_1 x_3$. With this order as a starting-point we may write down, as in Table XI, the preliminary values for the various factors entering into each variable.

As we attempt to explain more exactly the total situation represented by the nine variables we are immediately at a loss because of the insufficiency of our statistical technique to deal with so many

TABLE X

Means, Standard Deviations, and Correlations for Seventh-Grade Population of 140

Note.—Figures above the upper-left to lower-right diagonal are correlation coefficients corrected for attenuation; figures along this diagonal are reliability coefficients; figures below it are raw correlation coefficients.

	1	2	3	4	5	6	7	8	9
	Reading		Arithmetic		Memory				Space
	Speed	Power	Speed	Power	Words	Numbers	Meaningful Symbols	Meaningless Symbols	Power
σ	20.52	31.57	32.05	16.77	16.91	3.34	17.14	23.04	24.30
M	93.06	194.53	147.39	136.11	112.53	19.74	97.87	62.00	75.83
Reading—									
1. Speed	**.9197**	.6978	.2639	.0814	.2155	.3053	.3354	.2290	.2183
2. Power	.6328	**.8942**	—.0614	.0922	.3471	.3991	.3807	.2905	.3128
Arithmetic—									
3. Speed	.2412	—.0553	**.9083**	.5935	.0467	.3008	.0244	.0660	.0281
4. Power	.0586	.0655	.4248	**.5639**	.2099	.4865	.3592	.4159	.5335
Memory—									
5. Words	.1950	.3097	.0420	.1487	**.8903**	.9596	.7686	.5427	.4935
6. Numbers	.2318	.2988	.2270	.2893	.7169	**.6270**	.8647	.7005	.5869
7. Meaningful symbols	.2969	.3322	.0215	.2489	.6693	.6319	**.8517**	.8230	.6287
8. Meaningless symbols	.1999	.2501	.0573	.2843	.4662	.5050	.6915	**.8289**	.6404
Space—									
9. Power	.1960	.2770	.0251	.3751	.4361	.4351	.5433	.5459	**.8768**

variables at a single time. To meet the situation we will here make an assumption which will be either substantiated or discredited by subsequent steps. In brief, if this assumption is unsound it will so prove itself in the development and will not be perpetuated because of its entrance at this point. The assumption is that the thing which constitutes the general factor when variables x_1, x_2, x_3, x_4 are studied is the same thing as that constituting the general factor when variables x_1, x_2, x_3, x_5 are investigated, etc., for all other combinations of the nine variables, four at a time. Variables can be built up such that one general factor runs through x_1, x_2, x_3, x_4 and a second quite different general factor through x_1, x_2, x_3, x_5; and, as is of course obvious, situations can readily exist where the thing that is the general factor in x_1, x_2, x_3, x_4 is also the general factor in x_1, x_2, x_3, x_5. None of the criteria herein given dealing with tetrads reveals whether the first or second situation obtains. Though light upon this issue is given by the pentad function, still a thoroughgoing utilization of this criterion would be difficult because of the labor involved in obtaining necessary probable errors and it would simply make the issue a six-variable issue instead of a five-variable one; for we do not have, and certainly cannot readily obtain, criteria for six, seven, or more variables at a time. We will then assume that the general factor found in every set of four variables taken from the nine (in certain of these situations there is more than one general factor) is the same throughout, so that every variable may be expressed as is x_1 following

$$x_1 = c_1 a + \text{(other special or general factors)} + \text{(a specific factor not chance)} + \text{(a chance specific factor)} \quad \ldots\ldots [69]$$

In this equation c_1 is a constant and a is a variable changing from individual to individual, but remaining the same for a given individual throughout $x_1, x_2, x_3, \ldots x_9$. In the determination of "other special or general factors" we shall be guided by the findings already reported, namely, that there exists a very strong $x_1 x_2$ bond, a strong $x_3 x_4$ bond, etc.

If when so doing we run into a situation wherein the a factor and the special factors attributed to a certain variable lead to an unreasonably high reliability coefficient of the variable, we shall take this into consideration. What is an unreasonably high reliability coefficient is not obvious, but from rather sporadic investigations the writer believes that when the specific factors in mental tests are thought of as being less than five per cent of the total non-chance factors, the view tends to become unreasonable. In illustration: the reliability coefficient of x_1 was found to equal .92. If

$$x_1 = c_1 a + k_1 b + q_1 f + E_1 + e_1 \quad \ldots\ldots\ldots [70]$$

in which a, b, and f are general or special factors, E_1 a specific non-chance factor, and e_1 a chance factor, then the reliability coefficient is given by

$$r_{1I} = c^2{}_1\, \sigma^2{}_a + k^2{}_1\, \sigma^2{}_b + q^2{}_1\, \sigma^2{}_f + \sigma^2{}_{E_1} \quad \ldots\ldots\ldots [71]$$

or

$$c^2{}_1\, \sigma^2{}_a + k^2{}_1\, \sigma^2{}_b + q^2{}_1\, \sigma^2{}_f = r_{1I} - \sigma^2{}_{E_1} \quad \ldots\ldots\ldots [72]$$

It is here taken that $\sigma^2{}_{E_1}$ will hardly be less than $.05 r_{1I}$. Thus when the left-hand member exceeds $.95 r_{1I}$ the situation tends to become unreasonable. If for any variable the left-hand member does not exceed .95 of the reliability coefficient no use is made of the reliability coefficient in the subsequent treatment. Of the nine possible cases only three tend to become such that the reliability coefficient throws light upon the number and nature of the factors which are present. We may thus lay out Table XI in which it was expected from the analysis made and not here described that there would be entries in all the singly starred cells.

Double stars are to be found in five of the cells. As established by the preliminary analysis the necessity for entries in these five cells was not apparent, but later steps brought out this need. The reasons the preceding analysis did not discover the necessity for these entries are: (a) the entries are small in magnitude; (b) three of them are negative as shown in Table XII, thus weakening other positive bonds rather than standing out unequivocally on their own account; and (c) one of them, that in cell $d - x_2$, though positive, leads to an $x_2 x_5$ bond. Such a bond definitely discovered was thought to be represented by the b factor entering into x_2 and x_5, and it was not until it was established that one of these *b* factors (see β_5 of Table XII) was negative that the necessity for a *d* factor in x_2 came to light. That but five extra entries in Table XI are required to obtain, as will be shown, substantial adequacy in the postulation of factors underlying the nine variables is noteworthy. It shows a very satisfactory degree of accuracy in the preceding analysis as well as its inadequacy in giving certain details.

TABLE XI

PRELIMINARY GENERAL FACTORS

Variables	*a* Factor: Maturity, Race, Sex, Hetero-geneity, etc.	*b* Factor: Facility with Verbal Material	*c* Factor: Facility with Quantitative Concepts	*d* Factor: Memory	*e* Factor: Facility with Spatial Concepts	*f* Factor: Speed
x_1	* .3	* .7			* *	* .2
x_2	* .4	* .7		* *		
x_3	* .2	* *	* .6	* *		* .3
x_4	* .2		* .5		* .1	
x_5	* .5	* .1		* .5		
x_6	* .5		* .1	* .4		
x_7	* .5			* .3	* .2	
x_8	* .5			* .3	* .4	
x_9	* .5		* .1		* .4	* *

The numerical entries in the cells represent the first estimates by the writer of the standard deviations of the special factors. Using these preliminary estimates the reader will know that the special $x_1 x_2$ bond is represented by a numerical strength of .49 (given by $.7 \times .7$); that the special $x_3 x_4$ bond has a strength of .30; that the $x_5 x_6$ bond has a strength of .20; that the $x_7 x_8$ bond has a strength of .17 (given by $.3 \times .3$ plus $.2 \times .4$); that the $x_8 x_9$ bond has the strength of .16, and that the $x_1 x_3$ bond has the strength of .06. There is also indicated an $x_2 x_5$ bond of .07 (in harmony with the total 32, of column 25, Table VI); and an $x_1 x_5$

TABLE XII

Factor Values

Giving the relative standard deviations of general, group, and specific factors required to account for the correlations yielded by the nine tests given to the seventh-grade population of 140.

Tests	General Factor: Heterogeneity, Maturity, Sex, and Race α	Verbal Factor β	Number Factor γ	Memory Factor δ	Spatial Factor ϵ	Speed Factor ζ	Specific Factors: (Not Chance)	Specific Factors: (Chance)
Reading—								
1. Speed	.40	.69			.09	.38	.36	.28
2. Power	.38	.70		.20			.47	.33
Arithmetic—								
3. Speed	.23	—.15	.60	—.17		.66	.03	.30
4. Power	.21		.63		.31		.16	.66
Memory—								
5. Words	.66	—.09		.56			.36	.33
6. Numbers	.62		.25	.46			$\sqrt{-.03}$	.61
7. Meaningful symbols	.59			.52	.36		.32	.39
8. Meaningless symbols	.39			.46	.64		.24	.41
Space								
9. Power	.65		.14		.48	—.31	.33	.35

bond of .07 (not in harmony with the total 0, of column 15, Table VI); and an $x_3 x_6$ bond of .06 (not quite large enough to be in harmony with the total 60, of column 36, Table VI); an $x_3 x_9$ bond of .06 (not clearly indicated in Table VI); an $x_4 x_9$ bond of .09 (given by .5 × .1 plus .1 × .4, and indicated in Table VI). The remaining bonds, $x_5 x_6$, $x_5 x_7$, $x_5 x_8$, $x_6 x_7$, $x_6 x_8$, $x_6 x_9$, $x_7 x_9$, and $x_8 x_9$, are all more or less clearly indicated in Table VI. Utilizing these preliminary estimates as points of departure in making corrections to them, we obtain, as explained in chapter iii, the final values of Table XII.

Each entry in the "Specific Factors: (Not Chance)" column is such that combined with the other factors for the given variable it will just yield the obtained reliability coefficient. The specific factor for variable 6, $\sqrt{-.03}$, is of course an impossible value. To account for it, it is necessary to believe that the obtained reliability is smaller than the true value by .03 or a larger amount. Such a change in the reliability coefficient is not unlikely in view of its probable error, which is .035.

The adequacy of these values in the description of the total situation characterized by the correlations found is decidedly great, as shown in Table XIII. The distribution of discrepancies in the case of the inter-correlation coefficients, i.e., exclusive of the reliability coefficients, is given in Table XIV (page 107).

As a rough comparison to see if these divergences exceed what might be expected by chance, we may compare them with the probable error of the correlation coefficient. The median of the 36 coefficients is .285, and the corresponding probable errror is .05, because the population is 140. A comparison of the distributions of errors with the median probable error would seem to indicate that the fit was a remarkably good one, and in fact the writer believes it to be so, but he does not claim that the comparison is logically sound. It does not take into account the number of cells of Table XII filled with values, 31, in comparison with the number of independent equations of condition 39 (these come from the 36 inter-correlation coefficients and from reliability coefficients r_{3III}, r_{4IV}, and r_{6VI}).

TABLE XIII

TABLE OF ACTUAL AND THEORETICAL CORRELATIONS

$N = 140$

	Correlations Actually Found	Correlations Which Would Result from Such Factors as Postulated in Table XII	Differences
r_{12}	.63	.64	—.01
r_{13}	.24	.24	.00
r_{14}	.06	.11	—.05
r_{15}	.20	.20	.00
r_{16}	.23	.25	—.02
r_{17}	.30	.27	.03
r_{18}	.20	.21	—.01
r_{19}	.20	.19	.01
r_{23}	—.06	—.05	—.01
r_{24}	.07	.08	—.01
r_{25}	.31	.30	.01
r_{26}	.30	.33	—.03
r_{27}	.33	.33	.00
r_{28}	.25	.24	.01
r_{29}	.28	.25	.03
r_{34}	.42	.43	—.01
r_{35}	.04	.07	—.03
r_{36}	.23	.21	.02
r_{37}	.02	.05	—.03
r_{38}	.06	.01	.05
r_{39}	.03	.03	.00
r_{45}	.15	.14	.01
r_{46}	.29	.29	.00
r_{47}	.25	.24	.01
r_{48}	.28	.28	.00
r_{49}	.38	.37	.01
r_{56}	.72	.67	.05
r_{57}	.67	.68	—.01
r_{58}	.47	.52	—.05
r_{59}	.44	.43	.01
r_{67}	.63	.60	.03
r_{68}	.50	.45	.05
r_{69}	.44	.44	.00
r_{78}	.69	.70	—.01
r_{79}	.54	.56	—.02
r_{89}	.55	.56	—.01
r_{1I}	.92	Greater than .79	
r_{2II}	.89	Greater than .67	
r_{3III}	.91	Greater than .90	.01 !
r_{4IV}	.56	Greater than .54	.02 !
r_{5V}	.89	Greater than .76	
r_{6VI}	.63	Greater than .66	—.03 !
r_{7VII}	.85	Greater than .75	
r_{8VIII}	.83	Greater than .77	
r_{9IX}	.88	Greater than .77	

Let us now examine Table XII and its implications in some detail. It must be understood that the captions of the columns of Table XII describing the general and group factors α, β, γ, δ, ε, and ζ are merely words giving what the writer surmises to be the nature of the factors. Considering the β factor of Test 1, a more accurate statement of the case would be that there is an independent factor of standard deviation .69 (if the total standard deviation of

TABLE XIV

DISTRIBUTION OF DIFFERENCES BETWEEN ACTUAL AND THEORETICAL CORRELATIONS

Differences as given in Col. 4, Table XIII	−.05	−.04	−.03	−.02	−.01	.00	.01	.02	.03	.04	.05
Frequencies......	2	0	4	2	8	7	7	1	2	0	3

the variables is called 1.00) in this test which is also in Tests 2, 3, and 5. In Test 2 this factor has a standard deviation of .70, in Test 5 of .09, and in Test 3 of .15, but in this latter test it is an inhibiting factor, tending to decrease the test score, whereas in the other three tests it is a contributing factor tending the larger it is to increase the test score. Further, this same factor is not found[1] in Tests 4, 6, 7, 8, or 9. The appropriate verbal description of the factor can only be surmised by a study of the nine tests and the selection and naming of the element present in large amount in Tests 1 and 2, in slightly negative amount in Test 3, in slightly positive amount in Test 5, and absent in the remaining five tests. The writer's characterization of this as a "verbal" factor is the result of such a procedure on his part. If his analysis has been inadequate, the name which he has given to the factor is unfortunate, but the more important finding—that a β factor, having the

[1] This factor might be present in the other variables to a very small amount. In order to keep the number of cell entries below the number of independent conditions, it was planned that cell entries of less than .1 would be called .0. However, two of these entries turned out so nearly equal to .1 that they have been kept. They are the entries in the cells (x_1, ε) and (x_5, β).

correlation and independence properties listed, exists—is not weakened by the unhappy christening. That the factor present in large amount in x_1 and x_2 should be called "verbal" seems reasonable, but why it enters negatively in x_3 is not obvious. We may well await more evidence as to restraining and abetting mental functions and success in school subjects before attempting a general or comprehensive interpretation. However, in passing, the writer would say that the negative factor –.31, found in cell (x_9, ζ), though entirely unanticipated, seems strikingly in harmony with his a posteriori introspections as to what takes place mentally in the successful solution of the tasks of Test 9.

The γ factor, especially pronounced in x_3 and x_4, of some strength in x_6, and present in x_9, almost certainly has something to do with numbers in spite of its presence in x_9.

Tests 5, 6, 7, and 8 were devised with a view to ascertaining if there was a general memory factor. The δ factor is of substantial amount in all of these, of small amount in x_2, and it would seem in spite of the small negative factor in x_3 that this factor had much to do with memory. It thus seems that memory facility is a general factor operating with reference to words, numbers, and spatial material; in brief, operating in connection with all the material with reference to which tests were made. The claim that one is possessed of "memories" rather than a "memory ability" would seem not to be fully justified. There, of course, must be memories dependent upon the material involved, but the readiness with which this material can be memorized would seem to be general. This point should be further tested in additional studies by means of memory tests differing more radically in mechanical make-up and type of response than is the case with the four memory tests here employed.

The characterization of the ε factor as "facility in the manipulation of spatial relationships" might seem somewhat open to doubt in view of the fact that Test 9, especially devised to measure this trait, does not depend upon this factor to as great an extent as does Test 8. However, one should have anticipated that each of Tests 7, 8, and 9 would contain a spatial factor. One might not have anticipated this factor in Test 4, although its presence here is not at all inexplicable. Almost certainly one would not have anticipated a small factor of this sort in Test 1. However, in view of all these facts and of the fact later reported that this factor is also found in Test 10 (devised to measure speed in the manipulation of spatial relationships) in the case of another population, we may be justified in considering this factor as very definitely having much to do with the mental manipulation of spatial relationships. In the case of the small ε factor found in x_1 (Reading-Speed), and not found in x_2 (Reading-Power), may it not be that there is in rapid reading a visual stimulation and a corresponding spatial ideation of words and parts of sentences that is lacking in thought reading, and that it is of the nature of spatial manipulation? The value .09 is too small to be anything more than suggestive.

Three tests were devised to measure speed: Test 1—Reading-Speed; Test 3—Arithmetic-Speed; and Test 10—Speed in the Manipulation of Spatial Relationships. The ζ factor is found in x_1 and x_3 and in the case of another population in x_{10}. However, in this other population it is present as a negative factor. This would at first seem ridiculous, but from much evidence obtained at the time Test 10 was administered, it is known that this test is too hard to be a proper speed test in the seventh grade, and therefore it is in fact a power test. Thus the finding of a negative factor in Test 10 is in harmony with the finding of such a factor in Test 9. The writer believes that "speed" is a proper term to use in connection with this ζ factor.

An examination of the α factor discloses some highly interesting tendencies. It has already been pointed out that if a number of traits are genuinely independent, but if the subjects tested are dissimilar with respect to maturity, race, sex, and general nurture (i.e., nurture not affecting a single trait), a common factor due to this heterogeneity will appear. The factor has quite different values for the various tests; but if we postulate (*a*) that the individual differences due to maturity, race, and sex are less for tests most affected by specific nurture, i.e., reading and arithmetic, (*b*) that as has been found in various studies sex differences are of considerable amount with reference to memory; and (*c*) that maturity and perhaps sex differences find much expression in traits least affected by specific nurture, i.e., manipulation of spatial relationships, we have hypotheses which, with one exception (to be further considered), fairly well account for the variation in values of the α factor for the nine tests, and we have at least partial justification for designating this factor as due to heterogeneity in maturity, race, sex, and general nurture. By hypothesis (*a*) the factor for Tests 3 and 4, and to a less extent for Tests 1 and 2, should be low, and they are so found to be; by hypothesis (*b*) the factor for Tests 5, 6, 7, and 8 should be high, and this is the case except that α for Test 8 is somewhat too low; and by hypothesis (*c*) the α factor for Tests 7 and 8 should be high, as is the case except that the α factor for Test 8 is somewhat low. All the α factors except that for Test 8 fit in with these hypotheses, and one additional hypothesis makes Test 8 fit in. Let us postulate (*d*) that the α factor is really two factors, one of which is especially influenced by sex, being for memory of the nature "girls superior to boys," and for the manipulation of spatial relationships, of the nature "boys superior to girls." Then we find that the two tendencies work against each other in Test 8, and to a less degree in Test 7, and that no other test is affected by these tendencies jointly. This will decrease the α factor in these two instances, but it will decrease it more in the case of Test 8. Thus with this added postulation (*d*) all of the α entries are consistent.

This analysis of the α factor leads us to wonder whether, had we experimentally allowed for maturity, race, sex, and general nurture, any α or general factor would have remained. In other words, we may wonder if there is any factor at all independent of these things corresponding to Spearman's idea of a "central fund of intellective energy," or "general ability," or "*g*." To explain the α factor we have found it necessary to make postulations (*a*), (*b*), (*c*), and (*d*). Obviously further study should examine into each one of these. The data of the present study are not extensive enough to warrant such an investigation. The writer will also admit that the magnitude of the sex factor seems to be much greater than he anticipated. At the present time a recomputation treating the sexes separately is not possible. As this study is the first of its type, it is perhaps fortunate that a too accurate preliminary analysis was not made, for then the impression might be given that the final conclusions were merely expressions of preliminary impositions. Here with reference to sex, again (see chapter v) with reference to an η factor (possibly vivaciousness), and as has been shown with reference to a number of cells in Table XII, the statistical findings demand postulations not earlier made. The method is thus one of discovery as well as of proof. The next chapter reports a study similar to that of this chapter, covering more variables, but based upon a part only of the population used in this chapter.

CHAPTER V

THE FACTORS FOUND IN THE SEVENTH-GRADE POPULATION OF 109

Four additional test measures are available for 109 of the population of 140 studied in the last chapter. These measures are: Test 10, a test of speed of manipulation of spatial relationships; Test 11, the Wyman Free Association Test, scored to measure interest in intellectual activity; Test 12, the Wyman test, scored to measure interest in social activity; and Test 13, the Wyman test, scored to measure interest in physical activity. Test 10 is given in chapter viii. The Wyman list of stimulus words for which free associations were secured are not here given, because they would be of little utility. The important feature of the three Wyman interest measures is not the stimulus words, to which the pupils reacted with the first word that came to mind, but in the method of scoring these reactions; and the Wyman scoring scheme is very lengthy, as indeed it must be, for the number of reaction words which are given to a single stimulus word is very great and a separate score is necessary for each reaction word and for each type of interest. If the scoring scheme for a particular interest were here copied in full, it would be very difficult for the reader to ascertain by an examination of it what was the special nature of any unique factor in it. The writer well realized this difficulty at the time the test was included in the study, but these three measures were included because it was thought they might measure traits quite radically different from those measured by the other tests. They were included in order to discover experimentally if the same group and general factors required for the explanation of the ten tests would be sufficient to explain these tests as well, and, if not, to find out how many additional factors would be needed, rather than to determine with any certainty the nature of any additional factor or factors.

It should be said that the 109 were not quite a random sampling of the 140, as the following table shows:

TABLE XV

STATISTICS OF THE TWO SUB-SAMPLES OF THE POPULATION OF 140

Test No.	Means			Number of Population 140 Standard Deviations Apart of Means of Populations of 109 and 31
	Population of 140	Population of 109	Population of Remaining 31	
Reading—				
1. Speed	93.1	93.1	92.4	—.04
2. Power	194.5	197.3	184.6	—.40
Arithmetic—				
3. Speed	147.4	147.2	148.1	.03
4. Power	136.1	136.9	133.3	—.21
Memory—				
5. Words	112.5	113.3	109.7	—.21
6. Numbers ..	19.7	19.9	19.0	—.27
7. Meaningful symbols	97.9	98.4	96.1	—.13
8. Meaningless symbols	62.0	62.9	58.8	—.18
Space—				
9. Power	75.8	77.8	68.7	—.37

An examination of this table shows that except for the two speed measures, Tests 1 and 3, the 31 are about one-quarter of a standard deviation inferior to the 109. This may have some small systematic effect, but we should on the whole expect the same factor values for the population of 109 as for the population of 140, of which the 109 are a part.

The basic correlation data for the population of 109 are given in Table XVI.

Two entries in this table are exceptional. One is that giving the coefficient of correlation corrected for attenuation between Test 11, Intellectual Interest, and Test 12, Social Interest. This corrected coefficient is 1.13. Its excess over 1.00 can only be interpreted as

TABLE XVI

Means, Standard Deviations, and Correlations for Seventh-Grade Population of 109

Note.—Figures above the upper-left to lower-right diagonal are correlation coefficients corrected for attenuation; figures along this diagonal are reliability coefficients; figures below it are raw correlation coefficients.

	1	2	3	4	5	6	7	8	9	10	11	12	13
	Reading		Arithmetic		Memory				Space		Interest		
	Speed	Power	Speed	Power	Words	Numbers	Meaningful Symbols	Meaningless Symbols	Power	Speed	Intellectual	Social	Activity
σ	20.66	31.50	31.75	16.04	16.17	3.17	16.95	23.58	21.97	40.09	7.34	4.36	2.91
M	93.33	197.34	147.17	136.93	113.30	19.94	98.36	62.87	77.80	157.29	124.77	121.30	124.88
Reading—													
1. Speed	**.9192**	.7328	.2629	.0721	.2713	.3311	.3573	.2829	.2479	.2104	.4061	.4837	.0391
2. Power	.6637	**.8923**	.0435	.0885	.3341	.3179	.2929	.2716	.2866	.2429	.5660	.6473	.1631
Arithmetic—													
3. Speed	.2417	.0394	**.9196**	.6123	.0743	.3671	.1012	.1036	—.0153	.0176	—.0655	—.0043	.3070
4. Power	.0500	.0605	.4249	**.5236**	.2172	.4133	.4104	.4593	.4332	.4091	—.0654	.0912	.1451
Memory—													
5. Words	.2414	.2928	.0661	.1458	**.8610**	.9299	.7935	.5641	.4376	.3335	.3344	.3143	.1166
6. Numbers	.2490	.2355	.2761	.2346	.6768	**.6153**	.8613	.6711	.3646	.3437	.1390	.1612	.3123
7. Meaningful symbols	.3146	.2541	.0891	.2727	.6761	.6204	**.8433**	.8257	.5915	.4985	.1314	.1210	.0313
8. Meaningless symbols	.2493	.2358	.0913	.3055	.4811	.4838	.6969	**.8447**	.5756	.3736	.3031	.2875	.1268
Space—													
9. Power	.2203	.2509	—.0136	.2905	.3763	.2651	.5034	.4903	**.8589**	.8300	.0486	.0282	—.1430
10. Speed	.1859	.2115	.0156	.2728	.2852	.2485	.4219	.3165	.7090	**.8495**	.0525	—.0373	—.2437
Interest—													
11. Intellectual	.3421	.4698	—.0552	—.0416	.2726	.0958	.1060	.2448	.0396	.0425	**.7721**	1.1345	.2941
12. Social	.3477	.4584	—.0031	.0495	.2186	.0948	.0833	.1981	.0196	—.0258	.7474	**.5621**	.5408
13. Activity	.0245	.1008	.1927	.0687	.0708	.1603	.0188	.0763	—.0867	—.1470	.1691	.2653	**.4283**

the result of chance. In view of the size of the population it could quite reasonably be so attributed, but even to have a coefficient of correlation corrected for attenuation of 1.00 between these two variables is extreme, for earlier investigations upon entirely different populations have shown a substantial disparity between these two measures. However, for this particular population it is necessary both on account of this high coefficient corrected for attenuation and on account of factor values discovered, as will shortly be reported, to consider these two variables, x_{11} and x_{12}, to be practically identical.

The other surprising value is the correlation of .27 found between Tests 6 and 9, Memory Numbers and Space Power. The correlation between the same two variables for the population of 140 is .44. The writer has not attempted to determine the mathematical likelihood of getting, as a matter of chance, two correlations differing as much as do these two from populations of 140, and 109 drawn in a random manner from the 140. The likelihood must be extremely small. The computation of both these coefficients has been checked and rechecked in order to rule out the possibility of arithmetical error. It seems to the writer that the difference is an instance of an extreme divergence such as does occasionally occur as a matter of chance. Though it is disconcerting, it is only one of thirty-six correlations and may not seriously affect the general result coming from this population. We first require a table of tetrad differences. A sample only of such is given in Table XVII. The full table underlies further derived Tables XVIII, XIX, XX, and XXI.

From Table XVII we derive a table (on account of length not reproduced here), somewhat similar to Table VI of chapter iii, for all the variables, taken two at a time, in order to determine the strength of the bonds between each pair. The procedure pursued may be illustrated by considering the first entry of this unpublished table. Reference to Table XVII shows that $t_{1\,2\,3\,4} = .267$. Now $t_{1\,2\,3\,4} = r_{12}\,r_{34} - r_{13}\,r_{24}$. If this tetrad is positive it is due to a group bond between x_1 and x_2, or between x_3 and x_4, or to negative bonds between $x_1\,x_3$ or $x_2\,x_4$. As negative bonds will later be in-

TABLE XVII

TETRADS: SEVENTH-GRADE PUPILS—N=109

(First 33 only of the 715 rows in the full table.)

KEY

x_1=Reading Speed
x_2=Reading Power
x_3=Arithmetic Speed
x_4=Arithmetic Power
x_5=Memory—Words
x_6=Memory—Numbers
x_7=Memory—Meaningful Symbols
x_8 =Memory—Meaningless Symbols
x_9 =Space—Power
x_{10}=Spatial Manipulation Speed
x_{11}=Intellectual Interest
x_{12}=Social Interest
x_{13}=Activity Interest

Variables *abcd*	$r_{ab}r_{cd}$	$r_{ac}r_{bd}$	$r_{ad}r_{bc}$	t_{abcd}	t_{abdc}	t_{acdb}
1 2 3 4	.282	.015	.002	.267	.280	.013
1 2 3 5	.044	.071	.010	—.027	.034	.061
1 2 3 6	.183	.057	.010	.126	.173	.047
1 2 3 7	.059	.061	.012	—.002	.047	.049
1 2 3 8	.061	.057	.010	.004	.051	.047
1 2 3 9	—.009	.061	.009	—.070	—.018	.052
1 2 3 10..................	.010	.051	.007	—.041	.003	.044
1 2 3 11..................	—.037	.114	.013	—.152	—.050	.101
1 2 3 12..................	—.002	.111	.014	—.113	—.016	.097
1 2 3 13..................	.128	.024	.001	.104	.127	.023
1 2 4 5	.097	.015	.015	.082	.082	.000
1 2 4 6	.156	.012	.015	.144	.141	—.003
1 2 4 7	.181	.013	.019	.168	.162	—.006
1 2 4 8	.203	.012	.015	.191	.188	—.003
1 2 4 9	.193	.013	.013	.180	.180	.000
1 2 4 10..................	.181	.011	.011	.170	.170	.000
1 2 4 11..................	—.028	.023	.021	—.051	—.049	.002
1 2 4 12..................	.033	.023	.021	.010	.012	.002
1 2 4 13..................	.046	.005	.001	.041	.045	.004
1 2 5 6	.449	.057	.073	.392	.376	—.016
1 2 5 7	.449	.061	.092	.388	.357	—.031
1 2 5 8	.319	.057	.073	.262	.246	—.016
1 2 5 9	.250	.061	.065	.189	.185	—.004
1 2 5 10..................	.189	.051	.054	.138	.135	—.003
1 2 5 11..................	.181	.113	.100	.068	.081	.013
1 2 5 12..................	.145	.111	.102	.034	.043	.009
1 2 5 13..................	.047	.024	.007	.023	.040	.017
1 2 6 7	.412	.063	.074	.349	.338	—.011
1 2 6 8	.321	.059	.059	.262	.262	.000
1 2 6 9	.176	.062	.052	.114	.124	.010
1 2 6 10..................	.165	.053	.044	.112	.121	.009
1 2 6 11..................	.064	.117	.081	—.053	—.017	.036
1 2 6 12..................	.063	.114	.082	—.051	—.019	.032

ferred from an absence of positive bonds, we will not further consider these last alternatives. The positive tetrad value .267 is recorded in the table (not here given) under $x_1\ x_2$ and again under $x_3\ x_4$. A similar procedure is followed for all other tetrads.

If tetrad

$$t_{a\,b\,c\,d} \text{ is } \left\{ \begin{array}{l} \text{positive, bonds } x_a x_b, \text{ and again } x_c x_d \\ \text{negative, bonds } x_a x_c, \text{ and again } x_b x_d \end{array} \right\} \text{ are credited}$$

$$t_{a\,b\,d\,c} \text{ is } \left\{ \begin{array}{l} \text{positive, bonds } x_a x_b, \text{ and again } x_c x_d \\ \text{negative, bonds } x_a x_d, \text{ and again } x_b x_c \end{array} \right\} \text{ are credited}$$

$$t_{a\,c\,d\,b} \text{ is } \left\{ \begin{array}{l} \text{positive, bonds } x_a x_c, \text{ and again } x_b x_d \\ \text{negative, bonds } x_a x_d, \text{ and again } x_b x_c \end{array} \right\} \text{ are credited}$$

After crediting every tetrad twice to bonds according to this scheme, we have a new table of which Table XVIII is a summary.

For each bond the number of positive tetrads in excess of 55 gives us some rough indication of the number of times the bond between the two variables is not overshadowed by some other bond. A weak bond between two variables, if it does not come into conflict with other bonds, would lead to a large number of positive tetrads, though they might all be small. The sum of the positive tetrads gives us some rough indication of the size of the bond or bonds between the two variables. For example, for $x_1\ x_2$ we have 55 + 24 positive tetrads, and for $x_1\ x_3$ we have 55 + 26. But the sum for the positive tetrads for $x_1\ x_2$ is much greater than that for $x_1\ x_3$. We may surmise that the $x_1\ x_2$ bond is numerically larger than the $x_1\ x_3$ bond, but that it is operative only about the same number of times. The number of times that a bond is operative is approximately given by the number of times that the other variables taken two at a time do not involve the same factor as that giving rise to the bond in question.

From Table XVIII let us list the bonds roughly in the order of their magnitude, and, for all bonds involving any two of variables

TABLE XVIII

BASED UPON THE STRENGTH OF SPECIAL BONDS BETWEEN VARIABLES OF THE SEVENTH-GRADE POPULATION OF 109

NOTE.—If there were no special bonds the number of tetrads per each pair of variables would be 55.

Variables in Pairs	Number of Positive Tetrads*	Sum of All the Positive Tetrads
1,2	55+24	13.16
1,3	55+26	5.31
1,4	55—34	.26
1,5	55—14	1.40
1,6	55— 6	1.77
1,7	55+ 0	2.78
1,8	55—14	1.36
1,9	55+ 5	1.54
1,10	55+ 8	1.83
1,11	55+16	6.02
1,12	55+19	6.34
1,13	55—30	.80
2,3	55—10	.59
2,4	55—35	.32
2,5	55— 2	2.22
2,6	55—19	1.42
2,7	55—17	1.53
2,8	55—25	1.11
2,9	55+10	2.76
2,10	55+ 9	2.71
2,11	55+28	9.48
2,12	55+26	9.18
2,13	55+11	2.28
3,4	55+45	11.93
3,5	55—10	.94
3,6	55+26	6.09
3,7	55+ 2	1.33
3,8	55+ 5	1.35
3,9	55—33	.26
3,10	55—11	.54
3,11	55—48	.03
3,12	55—34	.14
3,13	55+45	5.86
4,5	55—17	1.27
4,6	55— 2	2.93
4,7	55+ 7	3.40
4,8	55+14	4.25
4,9	55+25	4.93
4,10	55+30	5.04
4,11	55—44	.17
4,12	55— 6	.61
4,13	55+16	1.77
5,6	55+18	10.41
5,7	55+12	7.88
5,8	55— 8	4.47
5,9	55+ 6	4.22
5,10	55— 1	3.33
5,11	55+15	3.99
5,12	55+ 6	2.99
5,13	55— 5	1.57
6,7	55+12	7.21
6,8	55— 8	4.76
6,9	55—10	2.64
6,10	55— 2	2.90
6,11	55—15	.95
6,12	55—14	.82
6,13	55+20	3.73
7,8	55+10	8.19
7,9	55+13	6.58
7,10	55+18	5.86
7,11	55— 9	1.00
7,12	55—19	.63
7,13	55—30	.94

* The sum of the entries in this column is 4,294. It should be 4,290. The difference comes from a slight error in the table not here given from which this table was derived. It was not considered of sufficient moment to warrant a redetermination of the faulty table.

TABLE XVIII—*Continued*

Variables in Pairs	Number of Positive Tetrads*	Sum of All the Positive Tetrads	Variables in Pairs	Number of Positive Tetrads*	Sum of All the Positive Tetrads
8,9	55+17	6.43	10,11.........	55— 5	.93
8,10.........	55+ 2	3.88	10,12.........	55—41	.21
8,11.........	55+ 8	3.36	10,13.........	55—51	.01
8,12.........	55+ 3	2.36			
8,13.........	55— 2	1.74	11,12.........	55+41	19.48
			11,13.........	55+29	4.50
9,10.........	55+45	15.70			
9,11.........	55—15	.70	12,13.........	55+40	7.37
9,12.........	55—21	.49			
9,13.........	55—41	.33			

* See footnote on p. 118.

2 to 9, indicate, if possible, the nature of the bond as already revealed by our study of the population of 140. In the second column of Table XIX, the nature of the bond is suggested very frequently with a question mark as uncertain. Of course any explanatory observation covering pairs of variables involving Tests 10, 11, 12, or 13 will be highly speculative. Certain weak bonds for variables 1 to 9 are suggested in Table XVIII for which no cause had previously been indicated. In such case, a question mark is entered in the second column of Table XIX.

We should now have a pretty good working idea of the bonds underlying the thirteen variables. From Tables XII and XIX preliminary estimates of the bonds for these variables were made as given in Table XX (page 121).

The steps already described in connection with the population of 140 were followed to secure improved estimates. The preliminary estimates did not prove to be very close, and it has required some sixty-two successive improved approximations to secure the final estimates of the following Table XXI, the entries of which with but few exceptions may be taken as accurate to the second decimal place. Here again it should be observed that the probable error of these figures is not known, but that it may well be in the neighborhood of .09.

Let us first note the differences between the cell entries in this

TABLE XIX

Variables in Pairs Approximately in the Order of Indicated Strength of Special Bonds—$N=109$

Variables—Positive Bonds	Suggested Nature of Bonds	Variables—Negative Bonds	Suggested Nature of Bonds
11, 12	Verbal ? *	10, 13	Spatial ?
9, 10	Spatial	3, 11	?
1, 2	Verbal *	4, 11	?
3, 4	Numbers	10, 12	?
5, 6	Memory	9, 13	Spatial ? Speed ?
2, 11	Verbal ? *	1, 4	? †
2, 12	Verbal ? *	2, 4	? †
3, 13	?	3, 12	?
12, 13	?	3, 9	Speed
3, 6	Numbers	1, 13	?
1, 3	Speed	7, 13	?
7, 10	Spatial ?	9, 12	?
4, 10	Spatial ?	7, 12	?
11, 13	?	2, 8	? †
1, 12	Verbal ? *	9, 11	?
4, 9	Numbers and Spatial	6, 12	?
7, 8	Memory and Spatial	6, 11	?
5, 7	Memory	4, 5	? †
6, 7	Memory	3, 10	?
8, 9	Spatial	2, 6	? †
7, 9	Spatial	2, 3	Verbal and Memory
1, 11	Verbal ? *	3, 5	Memory
6, 13	?	2, 7	? †
5, 11	?	7, 11	?
4, 8	Spatial	1, 8	? †
4, 13	?	1, 5	Memory ?
2, 9	? †	4, 12	?
8, 11	?		
4, 7	Spatial		
2, 10	?		
2, 13	?		
5, 9	? †		
5, 12	?		

* Since there are bonds between the following pairs of variables, $x_{11}x_{12}$, x_1x_{11}, x_2x_{11}, x_1x_{12}, x_2x_{12}, x_1x_2, and since this last is a β or verbal factor, there is probably a β bond in $x_{11}x_{12}$, x_1x_{11}, x_2x_{11}, x_1x_{12}, x_2x_{12}, although in some, or all, of these situations there are probably also other bonds operating.

† This pair of variables occurred in the study of the population of 140, but no bond of this nature was indicated in that study.

TABLE XX

PRELIMINARY ESTIMATES OF THE FACTORS ENTERING INTO THE THIRTEEN VARIABLES OF THE POPULATION OF 109 SEVENTH-GRADE CHILDREN

Tests	Maturity Heterogeneity α	Verbal β	Arithmetic γ	Memory δ	Spatial ε	Speed ζ	Additional Factor Especially in Tests 11, 12, 13 η
Reading—							
1. Speed ...	.40	.69			.09	.38	
2. Power ..	.38	.70		.20			
Arithmetic—							
3. Speed ...	.23	—.15	.60	—.17		.66	
4. Power ..	.21		.63		.31		
Memory—							
5. Words ..	.66	—.09		.56			
6. Numbers	.62		.25	.46			
7. Meaningful symbols	.59			.52	.36		
8. Meaningless symbols	.39			.46	.64		
Space—							
9. Power ..	.65		.14		.48	—.31	
10. Speed ...	.60				.60	—.20	
Interest—							
11. Intellectual	.30	.50	—.30		—.20		.65
12. Social ..	.20	.40	—.20		—.20		.55
13. Activity .	.15		.30		—.20	—.10	.30

table and those in Table XII, page 104, for the population of 140. The negative verbal factor value of –.09, Table XII, for Test 5, fell down to –.05, and was thus dropped entirely and does not

TABLE XXI

FINAL FACTOR VALUES FOR SEVENTH GRADE—$N = 109$

(Close of sixty-second successive approximation. Calculation error in third decimal place of about .005.)

Tests	Maturity Heterogeneity α	Verbal β	Arithmetic γ	Memory δ	Spatial ϵ	Speed ζ	Vivacity η
Reading—							
1. Speed ...	.426	.677	—.062		.109	.357	—.076
2. Power ..	.393	.726		.118			.069
Arithmetic—							
3. Speed ...	.252	—.077	.599	—.181		.643	
4. Power ..	.181		.610		.324		
Memory—							
5. Words ..	.677			.546			
6. Numbers	.855	—.067	.209	.453		.124	—.069
7. Meaningful symbols	.570		.066	.516	.388		—.132
8. Meaningless symbols	.385		.072	.480	.595	.084	.123
Space—							
9. Power ..	.582		.054		.477	—.316	—.176
10. Speed ...	.602			—.232	.457	—.287	—.188
Interest—							
11. Intellectual	.288	.422	—.147	.112			.724
12. Social ..	.231	.420		.092			.607
13. Activity .	.094		.238	.112	—.179	.076	.308

appear in Table XXI. A small negative verbal factor of –.07 for Test 6 here appears for the first time. A small positive numbers factor, .07, for Test 8, here appears. Small speed factors, .12 and .08, appear for variables 6 and 8. There have also developed small factors connecting six of the nine variables with the three interest tests. These are so small that they scarcely had an opportunity to manifest themselves before the interest tests were included. All of the additions and subtractions of factors mentioned, as well as other changes in amount, are of minor magnitude.

The addition of variables 10, 11, 12, and 13 has added much of interpretative value. First, the scores on the three interest tests are little influenced by the maturity or general factor; second, the intellectual and social interest measures have a substantial verbal element in them; third, the activity interest measure has a small number element and a small negative spatial factor; and finally, all three of the interest tests, though especially the social and intellectual interest tests, have a new factor in them. It is difficult to surmise what this new factor is, because of the nature and complexity of the scoring scheme of these tests. However, as the group upon which the scoring scheme was built was selected by means of the judgments of teachers, the factor is presumably something in children which tends to make teachers differentiate them upon the basis of their tendency to show interest in something or other—not interest in intellectual activity or social activity or physical activity, but interest in the abstract, for it must be something common to all three of these fields. The writer suggests that it may be described by the term *ebullience,* or perhaps *vivacity*. That it is found as a negative factor in the two spatial relationship tests suggests that it is opposed to mental deliberation, which from introspection would seem to be an asset in these two spatial relationship tests. Whatever it is, it appears to be quite unmeasured by the ordinary run of psychological tests, and thus to require much more study before it can be at all definitely described.

The test devised to measure speed of manipulation of spatial relationships did not accomplish this result, and except for a negative memory factor is very similar to the Spatial Power Test. This negative memory factor is interesting in that it suggests that in the tests involving such activity as the rapid turning over in the mind of geometrical forms a native tendency for a form to persist in the mind without change is a disadvantage.

The excellence with which the factors listed account for the correlation of the thirteen variables is pronounced as shown by accompanying Table XXII. For comparison with the differences shown, one should note that the probable error of the correlation coefficient of median size is .06.

TABLE XXII

ACTUAL AND THEORETICAL CORRELATIONS BETWEEN VARIABLES OF THE POPULATION OF 109 SEVENTH-GRADE CHILDREN—$N = 109$

Correlation Coefficients	Correlations Actually Found	Correlations Which Would Result from Such Factors as Postulated in Table XXI	Differences
$r_{1\,2}$	.66	.65	.01
$r_{1\,3}$	.24	.25	—.01
$r_{1\,4}$	.05	.07	—.02
$r_{1\,5}$	.24	.29	—.04
$r_{1\,6}$	.25	.24	.01
$r_{1\,7}$	.31	.29	.02
$r_{1\,8}$	.25	.25	.00
$r_{1\,9}$	.22	.20	.02
$r_{1\,10}$	.18	.22	—.04
$r_{1\,11}$	.34	.36	—.02
$r_{1\,12}$	.35	.34	.01
$r_{1\,13}$	.02	.01	.01
$r_{2\,3}$	.04	.02	.02
$r_{2\,4}$	.06	.07	—.01
$r_{2\,5}$	.29	.33	—.04
$r_{2\,6}$	.24	.23	.01
$r_{2\,7}$	.25	.28	—.03
$r_{2\,8}$	.24	.22	.02
$r_{2\,9}$	.25	.22	.03
$r_{2\,10}$	.21	.20	.01
$r_{2\,11}$	.47	.48	—.01
$r_{2\,12}$	.46	.45	.01
$r_{2\,13}$	.10	.07	.03
$r_{3\,4}$	.42	.41	.01
$r_{3\,5}$	.07	.07	.00
$r_{3\,6}$	.28	.28	.00
$r_{3\,7}$	.09	.09	.00
$r_{3\,8}$	.09	.11	—.02
$r_{3\,9}$	—.01	—.02	.01
$r_{3\,10}$	.02	.01	.01
$r_{3\,11}$	—.06	—.07	.01

TABLE XXII—*Continued*

Correlation Coefficients	Correlations Actually Found	Correlations Which Would Result from Such Factors as Postulated in Table XXI	Differences
$r_{3\ 12}$	.00	.01	—.01
$r_{3\ 13}$	.19	.19	.00
$r_{4\ 5}$	.14	.12	.02
$r_{4\ 6}$	.23	.23	.00
$r_{4\ 7}$	.27	.27	.00
$r_{4\ 8}$	.30	.31	—.01
$r_{4\ 9}$	.29	.29	.00
$r_{4\ 10}$	.27	.26	.01
$r_{4\ 11}$	—.04	—.04	.00
$r_{4\ 12}$	.05	.04	.01
$r_{4\ 13}$	.07	.10	—.03
$r_{5\ 6}$	.68	.64	.04
$r_{5\ 7}$	.68	.67	.01
$r_{5\ 8}$	.48	.52	—.04
$r_{5\ 9}$	.38	.39	—.01
$r_{5\ 10}$	.28	.28	.00
$r_{5\ 11}$	.27	.26	.01
$r_{5\ 12}$	.22	.21	.01
$r_{5\ 13}$	.07	.12	—.05
$r_{6\ 7}$	.62	.59	.03
$r_{6\ 8}$	.48	.46	.02
$r_{6\ 9}$	.26	.32	—.06
$r_{6\ 10}$	.25	.22	.03
$r_{6\ 11}$	.10	.11	—.01
$r_{6\ 12}$	.09	.11	—.02
$r_{6\ 13}$	.16	.14	.02
$r_{7\ 8}$	.70	.69	.01
$r_{7\ 9}$	.50	.54	—.04
$r_{7\ 10}$	.42	.43	—.01
$r_{7\ 11}$	.11	.12	—.01
$r_{7\ 12}$	.08	.10	—.02
$r_{7\ 13}$	.02	.02	.00
$r_{8\ 9}$	.49	.46	.03
$r_{8\ 10}$	.31	.35	—.04
$r_{8\ 11}$	.24	.24	.00
$r_{8\ 12}$	.20	.21	—.01
$r_{8\ 13}$	.08	.04	.04
$r_{9\ 10}$	.71	.69	.02
$r_{9\ 11}$	.04	.03	.01
$r_{9\ 12}$	.02	.03	—.01

TABLE XXII—*Concluded*

Correlation Coefficients	Correlations Actually Found	Correlations Which Would Result from Such Factors as Postulated in Table XXI	Differences
$r_{9\,13}$	—.09	—.10	.01
$r_{10\,11}$	.04	.01	.03
$r_{10\,12}$	—.02	.00	—.02
$r_{10\,13}$	—.15	—.13	—.03
$r_{11\,12}$	.75	.69	.06
$r_{11\,13}$	.17	.23	—.06
$r_{12\,13}$	.26	.22	.04
		Greater than the values which follow	
$r_{1\,1}$	.92	.79	
$r_{2\,2}$	.89	.70	
$r_{3\,3}$	.92	.87	
$r_{4\,4}$	.52	.51	.01!
$r_{5\,5}$	.86	.76	
$r_{6\,6}$	.61	.62	—.01!
$r_{7\,7}$	.84	.76	
$r_{8\,8}$	.84	.76	
$r_{9\,9}$	.86	.70	
$r_{10\,10}$	.85	.74	
$r_{11\,11}$	.77	.82	—.05!
$r_{12\,12}$	.56	.61	—.05!
$r_{13\,13}$	.43	.21	

As some of the test material proved too difficult to give to the third-grade group, the seventh-grade population just studied has provided us with more complete data than any other group. We will, however, in the next chapter, study the factors found in the third grade, particularly with a view to ascertaining if with children of this younger age the same independent mental traits are manifest as in the seventh grade.

CHAPTER VI

THE FACTORS FOUND IN THE THIRD-GRADE POPULATION

The first nine of the tests which were given to the seventh-grade pupils were within the range of the abilities of the third-grade children. In addition to these nine which were given to 110 pupils, one other test, No. 10, Manipulation of Spatial Relationships Speed, was given to the same group. However, only 60 of the 110 did sufficiently well on it to yield usable scores. Most of the fifty misunderstood directions entirely or at least did things of such a peculiar nature that it is uncertain what they were attempting to do. Thus for the select population of 60, drawn from the 110, we have data for ten tests. Let us first investigate the factors indicated by the population of 110. The basic correlations of the variables are given in Table XXIII.

Tetrad differences for all combinations of the variables four at a time are as given in Table XXIV.

From Table XXIV a third table was derived in which each tetrad difference was credited to the bonds which could conceivably give rise to it. This table is not here reproduced, but a summary of it is presented in Table XXV.

With Table XXV as a foundation, preliminary estimates of factor values were made as shown in Table XXVI. These preliminary estimates were altered, leading after fifty-four approximations to the final values as given in Table XXVII.

We may now compare the factor values found for these third-grade children with those previously found for the seventh-grade group. To facilitate this, Table XXVIII is given, in which the first entry in each cell is the factor value for the third-grade population, and the second entry is that for the seventh-grade group. Unfortunately we do not know the standard error of the differences between these values. It may reasonably, however, be in the neighborhood of .12.

We could measure the relationship between these two sets of

TABLE XXIII

Means, Standard Deviations, and Correlations for Third-Grade Population of 110

Note.—Figures above the upper-left to lower-right diagonal are correlation coefficients corrected for attenuation; figures along this diagonal are reliability coefficients; figures below it are raw correlation coefficients.

	1	2	3	4	5	6	7	8	9
	Reading		Arithmetic		Memory				Space
Tests	Speed	Power	Speed	Power	Words	Numbers	Meaning-ful Symbols	Meaning-less Symbols	Power
σ	19.55	30.39	22.66	17.20	18.52	3.87	10.19	11.64	21.22
M	47.85	76.68	83.95	60.18	63.09	11.47	37.54	21.70	42.82
Reading—									
1. Speed	**.9350**	.8064	.5102	.2904	.3257	.1996	.2234	.0054	.2387
2. Power	.7654	**.9636**	.3681	.4992	.5879	.4125	.3686	.1118	.3508
Arithmetic—									
3. Speed	.4413	.3232	**.8001**	.5374	.2994	.3610	.2758	.2767	.3033
4. Power	.2453	.4281	.4199	**.7631**	.4624	.4619	.3776	.3238	.5808
Memory—									
5. Words	.2905	.5322	.2470	.3725	**.8505**	.9049	.7062	.5312	.4819
6. Numbers	.1632	.3423	.2730	.3411	.7055	**.7147**	.8088	.5586	.4812
7. Meaningful symbols	.1811	.3033	.2068	.2765	.5459	.5731	**.7026**	.8257	.5635
8. Meaningless symbols	.0045	.0942	.2124	.2427	.4203	.4052	.5939	**.7363**	.5259
Space—									
9. Power	.2103	.3138	.2472	.4623	.4049	.3707	.4304	.4112	**.8303**

TABLE XXIV

TETRADS: THIRD-GRADE PUPILS—N=110

KEY

x_1=Reading Speed
x_2=Reading Power
x_3=Arithmetic Speed
x_4=Arithmetic Power
x_5=Memory—Words
x_6=Memory—Numbers
x_7=Memory—Meaningful Symbols
x_8=Memory—Meaningless Symbols
x_9=Spatial Manipulation Power

Variables a b c d	$r_{ab}\,r_{cd}$	$r_{ac}\,r_{bd}$	$r_{ad}\,r_{bc}$	$t_{a\,b\,c\,d}$	$t_{a\,b\,d\,c}$	$t_{a\,c\,d\,b}$
1 2 3 4	.321	.189	.079	.132	.242	.110
1 2 3 5	.189	.235	.094	—.046	.095	.141
1 2 3 6	.209	.151	.053	.059	.156	.098
1 2 3 7	.158	.134	.059	.025	.100	.075
1 2 3 8	.163	.042	.001	.121	.161	.040
1 2 3 9	.189	.138	.068	.051	.121	.070
1 2 4 5	.285	.131	.124	.155	.161	.006
1 2 4 6	.261	.084	.070	.177	.191	.014
1 2 4 7	.288	.074	.078	.214	.211	—.003
1 2 4 8	.186	.023	.002	.163	.184	.021
1 2 4 9	.354	.077	.090	.277	.264	—.013
1 2 5 6	.540	.099	.087	.441	.453	.012
1 2 5 7	.418	.088	.096	.330	.321	—.008
1 2 5 8	.322	.027	.002	.294	.319	.026
1 2 5 9	.310	.091	.112	.219	.198	—.021
1 2 6 7	.439	.049	.062	.389	.377	—.012
1 2 6 8	.310	.015	.002	.295	.309	.014
1 2 6 9	.284	.051	.072	.233	.212	—.021
1 2 7 8	.455	.017	.001	.438	.453	.016
1 2 7 9	.329	.057	.064	.273	.266	—.007
1 2 8 9	.315	.001	.020	.313	.295	—.018
1 3 4 5	.164	.061	.122	.104	.042	—.061
1 3 4 6	.151	.067	.069	.084	.082	—.001
1 3 4 7	.122	.051	.076	.071	.046	—.025
1 3 4 8	.107	.052	.002	.055	.105	.050
1 3 4 9	.204	.061	.088	.143	.116	—.028
1 3 5 6	.311	.079	.040	.232	.271	.039
1 3 5 7	.241	.060	.045	.181	.196	.015
1 3 5 8	.185	.062	.001	.124	.184	.061
1 3 5 9	.179	.072	.052	.107	.127	.020
1 3 6 7	.253	.034	.049	.219	.203	—.016
1 3 6 8	.179	.035	.001	.144	.178	.034
1 3 6 9	.164	.040	.057	.123	.106	—.017

TABLE XXIV—*Continued*

Variables a b c d	$r_{ab}\,r_{cd}$	$r_{ac}\,r_{bd}$	$r_{ad}\,r_{bc}$	t_{abcd}	t_{abdc}	t_{acdb}
1 3 7 8.........	.262	.038	.001	.224	.261	.038
1 3 7 9.........	.190	.045	.043	.145	.146	.001
1 3 8 9.........	.181	.001	.045	.180	.137	—.044
1 4 5 6.........	.173	.099	.061	.074	.112	.038
1 4 5 7.........	.134	.080	.067	.054	.066	.013
1 4 5 8.........	.103	.071	.002	.033	.101	.069
1 4 5 9.........	.099	.134	.078	—.035	.021	.056
1 4 6 7.........	.141	.045	.062	.095	.079	—.017
1 4 6 8.........	.099	.040	.002	.060	.098	.038
1 4 6 9.........	.091	.076	.072	.015	.019	.004
1 4 7 8.........	.146	.044	.001	.102	.144	.043
1 4 7 9.........	.106	.084	.058	.022	.047	.026
1 4 8 9.........	.101	.002	.051	.099	.050	—.049
1 5 6 7.........	.166	.089	.135	.077	.032	—.046
1 5 6 8.........	.118	.069	.003	.049	.115	.065
1 5 6 9.........	.108	.060	.148	.047	—.041	—.088
1 5 7 8.........	.173	.076	.002	.096	.170	.074
1 5 7 9.........	.125	.073	.115	.052	.010	—.042
1 5 8 9.........	.109	.002	.088	.118	.031	—.087
1 6 7 8.........	.097	.073	.003	.023	.094	.071
1 6 7 9.........	.070	.067	.121	.003	—.050	—.053
1 6 8 9.........	.067	.002	.085	.065	—.018	—.084
1 7 8 9.........	.074	.002	.125	.073	—.050	—.123
2 3 4 5.........	.120	.106	.223	.015	—.103	—.118
2 3 4 6.........	.110	.117	.144	—.007	—.033	—.027
2 3 4 7.........	.089	.089	.127	.001	—.038	—.039
2 3 4 8.........	.078	.091	.040	—.012	.039	.051
2 3 4 9.........	.149	.106	.132	.044	.018	—.026
2 3 5 6.........	.228	.145	.085	.083	.143	.061
2 3 5 7.........	.176	.110	.075	.066	.102	.035
2 3 5 8.........	.136	.113	.023	.023	.113	.090
2 3 5 9.........	.131	.132	.078	—.001	.053	.054
2 3 6 7.........	.185	.071	.083	.114	.102	—.012
2 3 6 8.........	.131	.073	.026	.058	.105	.047
2 3 6 9.........	.120	.085	.057	.035	.062	.027
2 3 7 8.........	.192	.064	.019	.127	.172	.045
2 3 7 9.........	.139	.075	.065	.064	.074	.010
2 3 8 9.........	.133	.023	.067	.110	.066	—.043
2 4 5 6.........	.302	.182	.128	.121	.175	.054
2 4 5 7.........	.234	.147	.113	.087	.121	.034
2 4 5 8.........	.180	.129	.035	.051	.145	.094
2 4 5 9.........	.173	.246	.117	—.073	.056	.129

TABLE XXIV—*Continued*

Variables *a b c d*	r_{ab} r_{cd}	r_{ac} r_{bd}	r_{ad} r_{bc}	$t_{a\,b\,c\,d}$	$t_{a\,b\,d\,c}$	$t_{a\,c\,d\,b}$
2 4 6 7.........	.245	.095	.103	.151	.142	—.009
2 4 6 8.........	.173	.083	.032	.090	.141	.051
2 4 6 9.........	.159	.158	.072	.000	.087	.086
2 4 7 8.........	.254	.074	.026	.181	.228	.048
2 4 7 9.........	.184	.140	.087	.044	.097	.053
2 4 8 9.........	.176	.044	.076	.132	.100	—.033
2 5 6 7.........	.305	.187	.214	.118	.091	—.027
2 5 6 8.........	.216	.144	.066	.072	.149	.077
2 5 6 9.........	.197	.139	.221	.058	—.024	—.083
2 5 7 8.........	.316	.127	.051	.189	.265	.076
2 5 7 9.........	.229	.123	.171	.106	.058	—.048
2 5 8 9.........	.219	.038	.132	.181	.087	—.094
2 6 7 8.........	.203	.123	.054	.080	.149	.069
2 6 7 9.........	.147	.112	.180	.035	—.032	—.067
2 6 8 9.........	.141	.035	.127	.106	.014	—.092
2 7 8 9.........	.125	.041	.186	.084	—.062	—.146
3 4 5 6.........	.296	.084	.102	.212	.195	—.017
3 4 5 7.........	.229	.135	.077	.094	.152	.058
3 4 5 8.........	.176	.060	.079	.117	.097	—.019
3 4 5 9.........	.170	.114	.092	.056	.078	.022
3 4 6 7.........	.241	.075	.071	.165	.170	.005
3 4 6 8.........	.170	.066	.072	.104	.097	—.006
3 4 6 9.........	.156	.126	.126	.029	.029	.000
3 4 7 8.........	.249	.050	.059	.199	.191	—.008
3 4 7 9.........	.181	.096	.068	.085	.112	.027
3 4 8 9.........	.173	.098	.060	.074	.113	.038
3 5 6 7.........	.142	.149	.146	—.007	—.004	.003
3 5 6 8.........	.100	.115	.150	—.015	—.050	—.035
3 5 6 9.........	.092	.111	.174	—.019	—.083	—.064
3 5 7 8.........	.147	.087	.116	.060	.031	—.029
3 5 7 9.........	.106	.084	.135	.023	—.029	—.051
3 5 8 9.........	.102	.086	.104	.016	—.002	—.018
3 6 7 8.........	.162	.084	.122	.078	.040	—.038
3 6 7 9.........	.117	.077	.142	.041	—.024	—.065
3 6 8 9.........	.112	.079	.100	.033	.012	—.021
3 7 8 9.........	.085	.091	.147	—.006	—.062	—.055
4 5 6 7.........	.213	.186	.195	.027	.018	—.009
4 5 6 8.........	.151	.143	.171	.008	—.020	—.028
4 5 6 9.........	.138	.138	.326	.000	—.188	—.188
4 5 7 8.........	.221	.116	.132	.105	.089	—.016
4 5 7 9.........	.160	.112	.252	.048	—.092	—.140
4 5 8 9.........	.291	.098	.194	.193	.097	—.096

TABLE XXIV—*Concluded*

Variables a b c d	$r_{ab}\,r_{cd}$	$r_{ac}\,r_{bd}$	$r_{ad}\,r_{bc}$	$t_{a\,b\,c\,d}$	$t_{a\,b\,d\,c}$	$t_{a\,c\,d\,b}$
4 6 7 8.........	.203	.112	.139	.091	.064	—.027
4 6 7 9.........	.147	.119	.265	.028	—.118	—.146
4 6 8 9.........	.140	.090	.187	.050	—.047	—.097
4 7 8 9.........	.114	.104	.275	.009	—.161	—.170
5 6 7 8.........	.419	.221	.241	.198	.178	—.020
5 6 7 9.........	.304	.202	.232	.101	.072	—.030
5 6 8 9.........	.290	.156	.164	.134	.126	—.008
5 7 8 9.........	.224	.181	.240	.044	—.016	—.050
6 7 8 9.........	.236	.174	.220	.061	.015	—.046

TABLE XXV

Number of Positive Tetrads, Based upon the Strength of Special Bonds between Variables of the Third-Grade Population of 110

Note.—If there were no special bonds the number of tetrads per each pair of variables would be 21. Numbers in parentheses indicate number of positive tetrads of value zero to first two decimal places.

Variables in Pairs	Number of Positive Tetrads	Sum of All the Positive Tetrads	Variables in Pairs	Number of Positive Tetrads	Sum of All the Positive Tetrads
1, 2.........	21+20	9.68	3, 7.........	21—16	.21
1, 3.........	21+16	4.91	3, 8.........	21+ 0	.97
1, 4.........	21+ 3 (1)	1.38	3, 9.........	21— 3 (2)	.74
1, 5.........	21+ 2	1.28	4, 5.........	21— 5 (1)	1.11
1, 6.........	21—11 (3)	.31	4, 6.........	21— 8	.89
1, 7.........	21— 7 (1)	.47	4, 7.........	21— 9 (2)	.67
1, 8.........	21—21	.00	4, 8.........	21— 4	1.06
1, 9.........	21— 2	.85	4, 9.........	21+ 8 (2)	2.89
2, 3.........	21+ 4 (2)	1.77	5, 6.........	21+12	4.30
2, 4.........	21+ 3 (1)	2.34	5, 7.........	21+ 2 (1)	2.27
2, 5.........	21+ 6	2.44	5, 8.........	21+ 2 (1)	2.21
2, 6.........	21— 4	.89	5, 9.........	21— 6 (1)	1.07
2, 7.........	21— 6 (1)	.60	6, 7.........	21+12 (1)	3.32
2, 8.........	21—15	.14	6, 8.........	21+ 5	2.59
2, 9.........	21— 8	.81	6, 9.........	21— 5 (2)	1.12
3, 4.........	21+12 (1)	3.13	7, 8.........	21+18	5.26
3, 5.........	21—13 (1)	.31	7, 9.........	21+ 6 (1)	2.36
3, 6.........	21+ 0 (3)	.71	8, 9.........	21+12	3.20

TABLE XXVI

PRELIMINARY ESTIMATES OF FACTORS ENTERING INTO THE NINE VARIABLES OF THE POPULATION OF 110 THIRD-GRADE CHILDREN

Tests	Maturity, Heterogeneity, etc. α	Verbal β	Arithmetic γ	Memory δ	Spatial ε	Speed ζ
Reading—						
1. Speed	.4	.7				.4
2. Power	.4	.7				
Arithmetic—						
3. Speed	.2	.1	.5			.6
4. Power	.2	.3	.6		.2	
Memory—						
5. Words	.6			.6		
6. Numbers	.5			.6		
7. Meaningful symbols	.5			.3	.5	
8. Meaningless symbols	.4	—.2			.6	
Space—						
9. Power	.5				.4	—.2

factor values by calculating the coefficient of correlation between them, were it not for the fact that our technique has required that a considerable number of cells in each table have zero values in them. There is thus an imposed tendency for zero values to be found. Keeping such pairs of zero factor values, the correlation between the two series is well above .90. Even omitting such pairs the correlation is .84 (Spearman ϱ value). It is thus quite safe to conclude that the intrinsic correlation between the two factors as found in the third grade and as found in the seventh grade is as high as .9. This is indeed a significant finding. It not only gives us confidence in the values found, but also suggests that the factors found are independent of the tutelage occurring between

TABLE XXVII

FINAL FACTOR VALUES FOR THIRD GRADE—$N=110$

NOTE.—Close of fifty-fourth successive approximation. Calculation error in third decimal place of about .005.

Variables	Maturity, Heterogeneity, etc. α	Verbal β	Arithmetic γ	Memory δ	Spatial ε	Speed ζ
Reading—						
1. Speed	.344	.613				.485
2. Power	.618	.755				.153
Arithmetic—						
3. Speed	.359		.361			.659
4. Power	.436	.196	.721		.126	
Memory—						
5. Words	.691	.116		.497	.109	
6. Numbers	.541		.139	.632	.189	
7. Meaningful symbols	.521			.296	.530	
8. Meaningless symbols	.550	−.317			.588	
Space—						
9. Power	.578		.224		.175	

grade three and grade seven, for, presumably, the third-grade children are not radically different from what the seventh-grade children were when they were in the third grade. Of course educational processes which have taken place between grades three and seven have affected the scores of the seventh-grade children. Such influence, however, seems to have been of a nature to raise the mean, but not to greatly change the idiosyncrasies of the pupils. This is diametrically opposed to Spearman's view as quoted in chapter i, page 19. It thus seems that the factors reported are established early in life. In fact it seems reasonable to attribute them, at least in major portion, to original nature.

TABLE XXVIII

FACTOR VALUES FOR THIRD- AND SEVENTH-GRADE PUPILS, RESPECTIVELY

Tests	Maturity, Heterogeneity, etc. α	Verbal β	Number γ	Memory δ	Spatial ε	Speed ζ
Reading —						
1. Speed	.34 .40	.61 .69			.09	.48 .38
2. Power	.62 .38	.76 .70		.20		.15
Arithmetic—						
3. Speed	.36 .23	–.15	.36 .60	–.17		.66 .66
4. Power	.44 .21	.20 –.09	.72 .63		.13 .31	
Memory—						
5. Words	.69 .66	.12		.50 .56	.11	
6. Numbers	.54 .62		.14 .25	.63 .46	.19	
7. Meaningful symbols	.52 .59			.30 .52	.53 .36	
8. Meaningless symbols	.55 .39	–.32		.46	.59 .64	
Space—						
9. Power	.58 .65		.22 .14		.18 .48	–.31

Let us examine Table XXVIII in more detail. The average of the maturity and other heterogeneity factors is slightly greater at the early age than at the later. This is in harmony with other evidence which commonly indicates a greater heterogeneity in the early elementary-school grades than in the later. It is particularly interesting to note that the heterogeneity factor has been decreased most in the Reading Power and Arithmetic Power tests, for these are the two subjects most depended upon for grading and promotion. We may surmise that if our group had not been a school group we would not have found a significant difference between the two groups in the general factor values in either the Reading Power or the Arithmetic Power tests.

The similarity of the two verbal factors in the two reading tests for the two grades seems to be conclusive evidence that such a factor exists and is of large amount. It should be given an independent position in matters of psychology and learning. That the verbal factor is large in Test 4, Arithmetic Power, in the third grade and not so in the seventh grade is not surprising, for in the third-grade arithmetic reasoning problems, even though couched in quite simple terms, are in fact much dependent upon reading ability. It is not obvious what is the occasion of the negative verbal factor in Test 8, Memory for Meaningless Symbols, in the third grade.

The consistency of the results bearing upon a number factor is impressive. Here again a psychology of learning based upon this single trait is needed.

The memory factor, though in the main revealing itself under the same conditions for the two populations, shows one substantial difference. The test entitled "Memory for Meaningless Symbols" does not seem to be a memory test at all for the younger group, but a spatial relationships test, and, as mentioned, a slightly inverse verbal ability test.

The manipulation of spatial relationships is clearly a large independent factor entitled to its own psychology. It probably does not play as large a part in school work as do the verbal, arithmetic, and memory factors, but it may play even a greater part than these three in adult activity represented by trades and engineering. It should certainly be studied in these connections.

The speed factor seems to be genuine enough in connection with those things, reading and arithmetic, in which it has probably been emphasized. To determine its significance in the various adult walks of life calls for careful study.

Bearing in mind that the smaller third-grade population, for which scores on ten tests are available, is a select population and one chosen in a manner which is difficult of precise definition, we may nevertheless find it suggestive to note the factor values found in the case of this group. Tables XXIX, XXX, XXXI, and XXXII are for this population similar to tables already given for the preceding populations.

TABLE XXIX

Means, Standard Deviations, and Correlations for Third-Grade Population of 60

Note.—Figures above the upper-left to lower-right diagonal are correlation coefficients corrected for attenuation; figures along this diagonal are reliability coefficients; figures below it are raw correlation coefficients.

Tests	1	2	3	4	5	6	7	8	9	10
	Reading		Arithmetic		Memory				Space	
	Speed	Power	Speed	Power	Words	Numbers	Meaning-ful Symbols	Meaning-less Symbols	Power	Speed
σ	17.50	29.39	22.90	15.39	18.82	3.79	8.45	10.68	18.30	33.58
M	48.00	79.33	85.80	63.00	67.17	12.13	39.20	22.90	48.92	109.67
Reading—										
1. Speed	**.9189**	.8337	.4961	.5088	.2691	.2121	.3849	.0092	.2138	.3346
2. Power	.7808	**.9546**	.3930	.4277	.5876	.4202	.3271	.1236	.3953	.3721
Arithmetic—										
3. Speed	.4313	.3482	**.8224**	.3078	.1967	.3844	.1509	.2015	.1229	.1919
4. Power	.4065	.3483	.2327	**.6948**	.3038	.3636	.1174	.1330	.5178	.3193
Memory—										
5. Words	.2371	.5278	.1640	.2328	**.8451**	.9689	.6879	.4503	.2737	.1212
6. Numbers	.1699	.3431	.2913	.2533	.7444	**.6984**	.8593	.5143	.2854	.0837
7. Meaningful symbols	.2798	.2424	.1038	.0742	.4796	.5446	**.5751**	.8999	.4738	.2202
8. Meaningless symbols	.0072	.0981	.1485	.0901	.3363	.3492	.5545	**.6602**	.3687	.3002
Space—										
9. Power	.1786	.3365	.0971	.3760	.2192	.2078	.3131	.2610	**.7592**	.8266
10. Speed	.2741	.3106	.1487	.2274	.0952	.0598	.1427	.2084	.6154	**.7302**

TABLE XXX

Based upon the Strength of Special Bonds between Variables of the Third-Grade Population of 60

Note.—If there were no special bonds the number of positive tetrads per each pair of variables would be 28. Numbers in parentheses indicate number of positive tetrads of value zero to first two decimal places.

Variables in Pairs	Number of Positive Tetrads	Sum of All the Positive Tetrads	Variables in Pairs	Number of Positive Tetrads	Sum of All the Positive Tetrads
1, 2.........	28+17 (1)	8.58	3, 10........	28— 2 (2)	.93
1, 3.........	28+20 (1)	4.72	4, 5.........	28— 3 (1)	1.05
1, 4.........	28+11 (2)	3.70	4, 6.........	28+ 1 (3)	1.37
1, 5.........	28— 8 (1)	.88	4, 7.........	28—25 (2)	.03
1, 6.........	28—13 (1)	.48	4, 8.........	28—12	.36
1, 7.........	28+ 1 (2)	1.49	4, 9.........	28+11	3.56
1, 8.........	28—28	.00	4, 10........	28+10 (2)	1.89
1, 9.........	28— 8 (2)	.71	5, 6.........	28+17	7.64
1, 10........	28+ 8 (3)	2.25	5, 7.........	28+ 9 (2)	3.85
2, 3.........	28+ 5 (2)	2.51	5, 8.........	28+ 5 (1)	2.76
2, 4.........	28+ 0	2.20	5, 9.........	28— 8 (2)	1.34
2, 5.........	28+10 (1)	3.36	5, 10........	28—17 (5)	.08
2, 6.........	28— 6 (2)	1.46	6, 7.........	28+16	5.01
2, 7.........	28— 6 (2)	.69	6, 8.........	28+11 (2)	3.10
2, 8.........	28—20	.19	6, 9.........	28— 9 (1)	.69
2, 9.........	28— 3 (2)	1.69	6, 10........	28—27 (1)	.00
2, 10........	28+ 3 (1)	2.01	7, 8.........	28+22 (1)	6.97
3, 4.........	28+ 7 (3)	2.06	7, 9.........	28+ 5 (3)	1.98
3, 5.........	28— 6 (3)	.51	7, 10........	28— 5	.83
3, 6.........	28+10	2.23	8, 9.........	28+10 (5)	2.22
3, 7.........	28—17 (2)	.17	8, 10........	28+11 (2)	2.00
3, 8.........	28+ 1 (2)	1.03	9, 10........	28+20	8.49
3, 9.........	28—18 (2)	.16			

From a study of Table XXX it is apparent that the process of selection of the sixty pupils has introduced a very great community of function between Tests 9 and 10. Whether this intimacy is one of manipulation of spatial relationships or of understanding directions or of something else we cannot say. It is, however, clear that a very substantial factor additional to the α, β, γ, δ, ε, ζ factors of the population of 110 third-grade children is required. Such a factor called ϑ (Spatial 2) has been introduced into the preliminary estimates of Table XXXI. Because of the

TABLE XXXI

Preliminary Estimates of the Factors Entering into the Population of 60 Third-Grade School Children

Tests	Maturity, Heterogeneity, etc. α	Verbal β	Number γ	Memory δ	Spatial 1 ε	Speed ζ	Spatial 2 θ
Reading—							
1. Speed	.35	.62				.49	
2. Power	.62	.75				.15	
Arithmetic—							
3. Speed	.36		.37			.66	
4. Power	.43	.20	.71		.14		
Memory—							
5. Words	.69	.12		.50	.11		
6. Numbers ..	.54		.14	.62	.18		
7. Meaningful symbols ...	.52			.30	.33		
8. Meaningless symbols	.55	—.32			.59		
Space—							
9. Power	.60		.23		.16		.50
10. Speed	.40	.20			.40		.50

uncertain nature of this factor, the ε factor, called spatial in the population of 110, is here designated "Spatial 1." From a study of the tests it would seem that Spatial 1 involves a sensing and retention of geometric forms, whereas Spatial 2 involves their manipulation. The final factor values as given in Table XXXII show much consistency, except for the factors of Tests 8 and 9, with the factor values for the larger third-grade population. On the whole these data add but little to those from the larger and less select third-grade population. Where the results of Table XXXII are not in harmony with those of Table XXVII the difference is

TABLE XXXII

Final Factor Values for Third Grade—N=60

Note.—Close of forty-fifth successive approximation. Calculation error in third decimal place about .005.

Variables	Maturity, Heterogeneity α	Verbal β	Arithmetic γ	Memory δ	Spatial 1 ϵ	Speed ζ	Spatial 2 θ
Reading—							
1. Speed	.374	.661	.133			.443	
2. Power	.614	.700				.199	
Arithmetic—							
3. Speed	.313		.251			.662	
4. Power	.333	.254	.609	—.107			.080
Memory—							
5. Words	.673	.122		.530		—.083	—.064
6. Numbers .	.548		.233	.701	.119		
7. Meaningful symbols ...	.513			.283	.432		
8. Meaningless symbols	.531	.313			.630		
Space—							
9. Power	.535		.187	—.155		—.131	.596
10. Speed	.319	.195		—.208	.153		.703

probably due to an error in the former table, because there is undoubtedly a large systematic error due to selection entering into the factor values for the population of 60, and there is also a much larger chance error in the data than in the data for the population of 110.

CHAPTER VII

THE FACTORS FOUND IN THE KINDERGARTEN POPULATION

The data herein utilized were collected and basic correlation and tetrad difference calculations made by Miss Guinevere Kotter (1926). They are reported in much greater detail in her master's thesis, "Mental Peculiarities of Kindergarten Children," on file in the Stanford University Library.

After having found that the independent mental traits were much the same for seventh-grade as for third-grade children, it seemed very desirable to examine still younger children, with a view to discovering if the same traits were present and independent at the younger age. The tests used for the third- and seventh-grade children were too difficult for the kindergarten children. So a number of very simple individual tests were devised. It was not feasible in the time available to attempt to devise tests which would parallel all of the tests given to the older children, but certain of these were quite closely paralleled, as is obvious from the following description of the tests.

Test 1, Memory for Meaningful Forms

1. circle
2. square
3. oblong
4. cross
5. quarter-moon or crescent
6. heart
7. triangle

A row of meaningful forms was shown and certain of them were touched by the experimenter in a standardized order. The row was then covered while at the same time a second row of forms was exposed. The child was called upon to point, in the proper order, to the forms in this second series which had been touched by the experimenter in the first row.

Test 2, Control of Meaningful Visual Memory Images

The same material was used as in Test 1; but here the child was given an opportunity to examine briefly the forms of the first row, and then the experimenter, instead of touching certain forms, removed certain ones while the child's eyes were covered. The child then looked at the remaining forms in the first row for a brief period. This row was then covered at the same time that the second row was exposed, and the child called upon to point to the forms which had been removed. There is thus a demand upon the subject that he not only secure an image of forms presented, but he must operate upon certain of the images, i.e., he must deduct from his concept those images, the causal forms of which still lie before him when his eyes are uncovered, and he must remember the forms which have been removed long enough to pick them out when the second row is presented. This task surely involves more than memory, and is referred to as a test of one's Control of Meaningful Visual Memory Images.

Test 3, Memory for Meaningless Forms

The same technique was here used as in the case of Test 1. The meaningless forms used were selected from specimens found in the third- and seventh-grade tests for memory of meaningless forms.

Test 4, Control of Meaningless Visual Memory Images

The material of Test 3 and the technique of Test 2 were employed in this test.

Test 5A, Memory for Meaningful Verbal Material

1. time	6. bring, in, get
2. may, arm	7. give, but, near, few
3. was, has	8. stop, on, fall, cold
4. care, some, run	9. true, go, came, spring
5. next, is, word	10. his, yet, why, late, come

Each series was given to the child orally with the instruction that he repeat it immediately afterward.

TEST 5B, MEMORY FOR MEANINGLESS VERBAL MATERIAL

1. bem
2. mif, het
3. nid, rul
4. tul, kek
5. pim, sul, riz
6. tek, zen, hap
7. var, pip, dop
8. rof, kul, tep, rel
9. mup, hif, lum, laz
10. fim, nud, pob, sep, meb

The administration of this test was similar to that of Test 5A.

TEST 6, DIVIDED FORMS TEST

1. circle
2. heart
3. oblong
4. square
5. cross
6. quarter-moon or crescent
7. triangle

The subject was shown a perfect form for five seconds and asked to "find two pieces that when put together will make this for us."

TEST 7, KNOX CUBE TEST

This test was given as standardized by Knox.

Tests 5A and 5B were divided with the intention of keeping them distinct in order to provide evidence as to a verbal bond, but the correlation for these two when corrected for attenuation was found to be 1.03 (P.E. = .034), thus indicating that the two tests measured the same function. This high correlation alone gives evidence of a verbal factor, and, most surprising, it locates the factor as something that is independent of meaning, for the meaningless words measure the selfsame function as do the meaningful words.

Certainly if when this verbal factor first appears in the kindergarten it is independent of "meaning," the thing which nurture has added, it is altogether reasonable to attribute this bond to original nature.

Because of this high correlation between Tests 5A and 5B, they were combined into a single Test 5, Memory for Verbal Material. Their combination at this stage of the study precludes the later re-establishment of a verbal factor unless such a factor is already present in one or more of Tests 1, 2, 3, 4, 6, 7. From the nature of these tests this does not seem very probable. The further study of the correlations should give us evidence as to memory and spatial relationship factors. The presumable presence of a memory factor in at least six (Tests 1, 2, 3, 4, 5, 7) of the seven tests mitigates against its clear-cut discovery, for it almost partakes of the nature of a general factor for this group of tests.

Originally, three tests involving numbers were planned, but the time demands when giving individual tests are so great that it was necessary to omit these. We thus have no means of discovering the presence of a number factor. We also have no means of discovering a speed factor or an ebullience factor. Table XXXIII following and Table XXXIV (page 146) give the basic data.

TABLE XXXIII

Means, Standard Deviations, and Correlations for 107 Kindergarten Children of Ages 3 Years 6 Months to 6 Years 3 Months

Note.—Figures above the upper-left to lower-right diagonal are correlation coefficients corrected for attenuation; figures along this diagonal are reliability coefficients; figures below it are raw correlation coefficients.

	1	2	3	4	5	6	7
σ	1.75	1.65	1.88	1.80	2.68	2.03	2.15
M	7.00	6.98	7.11	7.12	9.84	7.21	6.46
1. Memory for Meaningful Forms	**.6850**	.8826	.6603	77.61	.5814	.6543	.9663
2. Control of Meaningful Verbal Memory Images	.5490	**.5650**	.8802	.5755	.4934	.6706	.8295
3. Memory for Meaningless Forms	.4678	.5666	**.7330**	.7276	.5265	.8573	.8517
4. Control of Meaningless Visual Memory Images	.4974	.3351	.4825	**.6000**	.3657	.4951	.7603
5. Memory for Verbal Material	.4499	.3469	.4215	.2649	**.8740**	.2924	.6939
6. Divided Forms	.4427	.4123	.6003	.3136	.2236	**.6690**	.7013
7. Knox Cube Test	.5741	.4478	.5237	.4229	.4660	.4118	**.5160**

Table XXXIV indicates a number of bonds. We can better judge just how many if we first calculate at least one standard error of a mean tetrad. For example there are eleven tetrads indicating a positive x_1x_2 bond. This is one more than chance would yield. The mean of all the twenty tetrads is .02, which is .02 more than chance would yield. Is this departure from chance significant? The standard error of this mean tetrad is .032. As the mean tetrad is only two-thirds its standard error, we do not feel called upon to explain an x_1x_2 bond.

Let us now examine the mean x_1x_3 tetrad. Its value is —.09, and its standard error is .042, so that the mean tetrad is over twice its standard error. We must therefore consider that there is a real negative bond of some sort between the two variables x_1 and x_3.

Preliminary estimates of factor values were made and corrected by successive approximations, resulting in Table XXXV. The procedure was the same as that employed with earlier populations except that in this case all the reliability coefficients were used. Each was multiplied by .95, and the endeavor was then made to secure factor values that would account for all the inter-correlations and these reduced reliability functions. The adequacy with which the factor values of Table XXXV account for the obtained inter-correlations and reliability coefficients is decidedly great.

The general maturity and heterogeneity factor is greater than in the case of the third- and seventh-grade groups. This is as one would expect since the primary group includes a wide age range, especially when expressed as a percentage of the mean age, and such percentage expression is probably fairly reasonable when dealing with the maturity of young children. Further, the members of the group have been in school so short a time that they have not as yet been sorted out on the basis of ability. At this level such a sorting would largely be on the ground of maturity and of racial differences.

As anticipated, a factor which we may call the verbal factor is very large in Test 5, Memory for Verbal Material, and scarcely present in any of the other tests.

TABLE XXXIV

Tetrad Differences Allocated to the Pairs of Variables from Which They Conceivably Could Have Arisen in the Population of 107 Kindergarten Children

1, 2	1, 3	1, 4	1, 5	1, 6	1, 7	2, 3	2, 4	2, 5	2, 6	2, 7
.11	—.12	.02	.09	.03	.12	.02	.01	.02	.00	.03
.06	—.09	.12	.02	.06	.04	.12	.00	.03	.06	.00
.08	—.06	.02	.01	.07	.04	.09	.02	.03	.03	.04
.14	—.12	.02	.03	.08	.01	.02	.04	.03	.04	.01
.08	—.11	.06	.00	.04	.02	.06	.01	.02	.01	.04
.02	—.06	.03	.06	.01	.08	.12	—.12	.03	.02	.05
.04	—.14	.03	.09	.01	.01	.04	—.11	.08	.06	.00
.01	—.08	.09	.08	.08	.02	.01	—.02	.03	.02	—.12
.06	—.09	.08	.17	.02	.11	.02	—.06	.03	.05	—.08
.05	—.09	.15	.02	—.08	.15	.06	—.02	.04	.03	—.01
.03	—.07	.06	.02	—.06	—.03	.08	—.03	—.09	.03	—.05
—.02	—.15	.08	.03	—.02	—.04	.08	—.04	—.06	.04	—.04
—.02	—.08	.04	.04	—.03	—.00	.02	—.03	—.03	.09	—.03
—.04	—.06	.02	.06	—.04	—.06	—.08	—.01	—.00	.05	—.02
—.01	—.08	.02	.01	—.03	—.04	—.02	—.00	—.06	—.06	—.08
—.03	—.17	—.01	—.05	—.08	—.08	—.02	—.04	—.01	—.14	—.03
—.02	—.02	—.01	—.01	—.08	—.01	—.02	—.06	—.08	—.00	—.04
—.03	—.02	—.02	—.04	—.11	—.02	—.05	—.05	—.00	—.03	—.03
—.06	—.15	—.01	—.02	—.02	—.08	—.08	—.03	—.01	—.05	—.09
—.01	—.04	—.03	—.02	—.02	—.06	—.04	—.00	—.05	—.02	—.04
Sums .44	—1.80	.93	.59	—.17	.18	.43	—.52	—.05	.31	.49
Means .02	—.09	.05	.03	—.01	.01	.02	—.03	.00	.02	.02

TABLE XXXIV—*Continued*

3, 4	3, 5	3, 6	3, 7	4, 5	4, 6	4, 7	5, 6	5, 7	6, 7
.11	.06	.08	.08	.01	.02	.04	—.03	.06	.03
.01	.01	.14	.06	.01	.02	.01	—.06	.05	.02
.09	.09	.08	.02	.00	.03	.04	—.08	.08	.02
.07	.08	.15	.04	.03	.03	.01	—.17	.04	.06
.02	.01	.08	—.04	.03	.00	.02	—.01	.08	.02
.08	.02	.17	—.02	.05	.02	.06	—.03	.02	.04
.03	.05	.11	—.01	.01	.05	.03	—.05	.08	.03
.02	.02	.15	—.11	—.01	.04	.03	—.08	.08	.06
.02	.04	.00	—.04	—.03	—.02	.04	—.03	.04	.01
.04	.06	.02	—.06	—.09	—.07	.04	—.03	.01	—.01
.09	—.02	.03	—.01	—.09	—.15	.09	—.05	.09	—.15
.05	—.01	.08	—.08	—.02	—.01	.06	—.02	.05	—.04
.03	—.08	.05	—.05	—.04	—.02	—.08	—.16	.09	—.02
—.02	—.03	.04	—.02	—.08	—.02	—.06	—.06	.05	—.04
—.08	—.01	.03	—.04	—.02	—.02	—.04	—.05	.16	—.00
—.00	—.03	.05	—.09	—.03	—.00	—.02	—.01	.11	—.04
—.02	—.08	.09	—.09	—.04	—.03	—.01	—.08	.05	—.05
—.05	—.03	.06	—.03	—.04	—.03	—.05	—.06	.04	—.06
—.02	—.05	.16	—.16	—.09	—.09	—.05	—.09	—.02	—.11
—.06	—.11	.11	—.05	—.04	—.03	—.01	—.04	—.02	—.04
Sums .41	—.01	1.68	—.70	—.48	—.28	.15	—1.19	1.14	—.15
Means .02	.00	.08	—.04	—.02	—.01	.01	—.06	.06	—.01

A factor which we may surely designate as a memory factor is large in Tests 5, Memory for Verbal Material, and 7, Knox Cube Imitation Test, and is not negligible in Test 3, Memory for Meaningless Forms, or in Test 1, Memory for Meaningful Forms.

TABLE XXXV

FINAL FACTOR VALUES FOR THE POPULATION OF 107 KINDERGARTEN CHILDREN AT CLOSE OF 48TH SUCCESSIVE APPROXIMATION

Tests Used	Maturity, Heterogeneity, etc. α	Verbal β	Memory δ	Spatial No. 1 ϵ	Spatial No. 2 θ	Control of Meaningless Content? η
1. Memory for Meaningful Forms........	.79		.13			
2. Control of Meaningful Visual Memory Images	.71			.27		—.18
3. Memory for Meaningless Forms.......	.62		.18	.27	.50	
4. Control of Meaningless Visual Memory Images	.63				.15	.42
5. Memory for Verbal Material	.49	.61	.50			
6. Divided Forms Test	.52				.58	—.24
7. Knox Cube Test.....	.62		.36		.13	

Test 3, Memory for Meaningless Forms, and Test 6, Divided Forms Test, have a large common factor which it is reasonable to designate as spatial. Traces of this same factor are found in Test 4, Control of Meaningless Visual Memory Images, and Test 7, Knox Cube Test.

There is a small factor entering into Tests 2 and 3, which likewise must be spatial. The existence of such a factor was entirely unanticipated, but a study of Spatial 1 and Spatial 2 of the third-grade population immediately suggests that this x_2x_3 factor is what was earlier called Spatial 1 (sensing and retention of geometric forms), while the larger factor found in Tests 3 and 6 and to a small extent in Tests 4 and 7 is what was called Spatial 2 (manipulation of spatial relationships). The factor values for this kindergarten population were determined by the writer and his assistants without knowledge of what the variables stood for, and it was both surprising and assuring when the variables were finally labeled to see how beautifully the factors conformed to those already discovered in the third- and seventh-grade populations. There is but a single factor found here not foreshadowed by the earlier work.

There is a large factor in Test 4, Control of Meaningless Visual Memory Images, which is in the main specific to this test. Indications of the same factor, though in a negative sense, are found in Test 2, Control of Meaningful Visual Memory Images, and Test 6, Divided Forms Test. Tentatively, this factor is referred to as a "Control of Meaningless Content" factor, but not having evidence of a similar factor in the third- and seventh-grade populations, the fitness of this designation is very much open to question.

In brief, all the factors except this last one seem to be of the same nature as factors revealed by the older populations. That the following traits, (1) facility with verbal material, (2) manipulation of spatial relationships, (3) memory, are independent categories of mental life from a very early age (probably from birth) seems hardly open to question. The sensing and retention of geometric forms should probably also be included in this list, also a number factor, because of its indubitable presence in the third- and seventh-grade populations, for it should be recalled that no test for this factor was made in the case of the kindergarten group.

CHAPTER VIII

DESCRIPTION, SCORING, AND RELIABILITY OF TESTS USED

Samples of all new tests used are provided below. The test designated "Reading Exercises" is the Reading Speed test, Test 1 (pages 166–68, 180–81). The Stanford Paragraph Meaning Test is the Reading Power test, Test 2 (for description of this and of Arithmetic Power, see *Stanford Achievement Tests,* World Book Company, 1926). The test designated "Arithmetic Exercises" is the Arithmetic Speed test, Test 3 (pages 169–70, 182–83); and the Stanford Arithmetic Reasoning Test is the Arithmetic Power test, Test 4. The test designated "Circles Test" has four types of material (pages 174–77, 186–89): that part comprising verbal material is the Memory Verbal test, Test 5; that part comprising numbers is the Memory Numbers test, Test 6; that part comprising meaningful symbols is the Memory for Meaningful Symbols test, Test 7; and that part comprising meaningless symbols is the Memory for Meaningless Symbols test, Test 8. The one-page test which is designated "Figure Exercise No. 2" (pages 178–79, 190) is the Manipulation of Spatial Relationships Power test, Test 9. And the two-page test designated "Figure Exercise No. 1" (pages 171–73, 184–85) is the Manipulation of Spatial Relationship Speed test, Test 10.

The directions as here given (taken, when shown in smaller type, from stenographic report of examiner's words) were rather closely followed in both the third and the seventh grades, except that in the third grade about twice as much foretest and preliminary training was given in the case of the figure exercises and a more mature mode of expression was at times used in the seventh grade than in the third grade. Some abridgment of these instructions was made the second day when Form B of the tests was given. Judging by the test results there was no case in which a child understood the directions on the first day and failed to understand them on the second.

All but one of the tests are well adapted to the seventh-grade level of ability, the exception being Test 10, which was only fairly satisfactory in this grade. All but the speed and power tests in spatial manipulation are well adapted to the third-grade level. In the third grade these two should have been preceded by fore-exercises more detailed than those actually given. For observations as to the nature of Tests 11, 12, and 13, see chapter v, page 112.

FORM A

Speed of Reading Test (Test 1).—Following are directions used in the third grade:

At the top of these sheets, where it says "name," write your name. Use blue pencils. Right below where it says "Name," it says "Age." Write the number that tells how old you are now. Right next to that it says, "When is your next birthday?" Write it down if you know it. If you don't know it, you will have to leave that blank. Under that it says, "How old will you be then?" Write the number that tells how old you will be on your next birthday. On the top line it says "Boy or girl." Write down which you are. Next to that it says "Date." What date is it today? [Give date.] Write down [date]. Under that it says "School." [Give name of school.] Write down [name of school]. Some of you are in the low third and some of you are in the high third. All of those in the low third write down "L 3" [illustrate on blackboard]. All of those in the high third write down "H 3."

On the first page there is a little story, "When the Train Passes." [Read directions on page 1, Form A.] A parenthesis looks like this [illustrate on blackboard]. Ready, go. Use blue pencils. [Pupils were provided with pencils having blue lead at one end and red at the other. By changing colors it was possible to score the test for different time limits.] [After five minutes] Stop. Now turn your pencils around and go right on. [Assistant must see that red lead is being used.] [After four minutes] Stop.

The five-minute score was used in preference to the nine-minute score because the nature of the material seemed to indicate that the latter was more of a "power" score for these third-grade children—the more speedy of them having finished the first story and got well into the second one. It was determined by the method explained in chapter iii, page 94, that the optimum scoring scheme was $(A - .8\,E)$ for Form A, and $(A - 1.3\,E)$ for Form B, in which A is the number of parentheses attempted and E the number incorrectly marked. The scoring scheme which was actually used was $(A - E)$ or the number of parentheses correctly marked in five minutes.

The reliability of the scores thus obtained is .878 for the population of 110 and .850 for the population of 60. As the actual score used is the sum of the scores on the two forms, these reliabilities were stepped up by the Spearman-Brown formula, giving the reliabilities of Tables XXIII and XXIX.

The time limits of the scoring scheme for this test were the same in the seventh grade as in the third grade. Although the optimum scoring scheme was $(A - 1.2\,E)$ for Form A, and $(A - 1.4\,E)$ for Form B, these were not sufficiently better scoring schemes over $(A - E)$ to justify using them. The reliability of the single form for the population of 140 is .851, and for the population of 109 it is .850. These stepped up give the reliabilities of Tables X and XVI.

Reading Power Test (Test 2).—In both the third and seventh grades the score is the sum of the scores on the three parts—paragraph meaning, sentence meaning, and word meaning—of the Stanford Achievement Test, which was given and scored as described in the 1926 *Manual of Directions*. Form A, Primary Examination, was used in the third grade, and Form A, Advanced Examination, in the seventh grade. The reliability for the third grade was obtained by splitting the test (all three parts), correlating their odds against the evens. In the paragraph meaning test, one paragraph, and not one blank, was considered an element when splitting into halves. These half-scores were correlated and then stepped up by the Spearman-Brown formula to obtain the reliabilities given in Tables XXIII and XXIX. In the seventh grade the reliabilities for the 140 and 109 populations were estimated from reliability data at hand covering a large normal population of fourteen-year-olds (see Kelley, 1926, *Influence*). The means for the normal population of fourteen-year-olds and for the present population of 140 seventh-grade children were 224 and 195, respectively. This is sufficiently close to warrant the assumption of an equal-standard error of estimate in the two cases. Making this assumption and using the formula connecting reliability with range examined (Formula 178 of Kelley, 1924, *Statistical*), yields the reliabilities of Tables X and XVI.

Arithmetic Speed Test (Test 3).—The directions for this test were as follows:

> Turn your books to the back page, page 8. [See page 173.] It says at the top "Computation Exercises." [Read directions aloud.] Turn to pages 4 and 5. Work the problems in order. Work as fast as you can, drawing a line under the right answer. If there is any one that you cannot do, leave it and go on to the ones after it. Work with your blue pencils.

Various working times were examined by the device of having the pupils work with differently colored pencils during different periods. The score finally adopted was based upon six minutes' working time in the third and five minutes' working time in the seventh grade. The optimum scoring scheme for the third grade was found to be $(S - 1.48\,E)$, in which S is the number of problems attempted and E the number of errors. With this score the reliability is .692. The scoring scheme actually used is $(S - E)$ or the number of problems correctly worked in six minutes. This has a reliability of .678 on the same population ($N = 122$), giving the reliability of .692 for the optimum scoring method. For the population of 110 the reliability is .667, and for the population of 60 it is .698. These last two were stepped up by the Spearman-Brown formula to give the reliabilities of Tables XXIII and XXIX.

The time limit for the seventh grade is five minutes. The optimum scoring scheme is $(S - 1.41\,E)$. The scheme actually used is $(S - E)$, which has a reliability for the population of 140 of .832, and for the population of 109 of .851. These were stepped up giving the reliabilities of Tables X and XVI.

Arithmetic Power Test (Test 4).—The test used to measure computation power was the Stanford Computation Test. It was given and scored as directed in the 1926 *Manual of Directions.* The reliabilities were determined by interpolation in a table of standard errors of individual scores not as yet published. It may be looked for in the next edition of the *Stanford Achievement Test Manual.* The reliabilities given in Tables XXIII, XXIX, X, and XVI are those for a single form of the test, which is what was used in measuring these pupils.

Memory tests: Test 5 for Verbal Material; Test 6 for Numerical Material; Test 7 for Meaningful Symbols; and Test 8 for Meaningless Symbols. The directions for the seventh grade were as follows:

Write your name, sex, age at last birthday, date, grade that you are in, and the name of your school, in the spaces provided. Today is the [give date] and you are in the [give grade], and of course this school is the [give name of school].

Now look at the squares in which are words, numbers, and symbols, beside each of which is a little star. I am going to ask you to draw lines connecting certain of these stars. Look at this card [point to the words "of," "to," "on," appearing on the face of Card No. 2] and fix these words in mind, so as to remember them [after ten seconds turn card over, exposing the number 2 on the back]. Now, find a square with the number 2 right in the middle of it, and as soon as you have found it, draw a line connecting the stars which are right by the side of the three words which you have just seen. Draw the line right now [allow ten seconds]. If you have done it correctly, it will look like this. [Show sample on an 18-inch square cardboard, and have assistants walk about the room to discover any who have it wrong. If any seem perplexed, do this over, explaining and watching those having difficulty.] Now, let us do another one. Watch carefully [show Card No. 11 for eight seconds, and then show the reverse, giving the number of the card]. This is Card No. 11, so find Square No. 11 and connect in the proper way the three numbers which you have seen [allow ten seconds]. If you have done it correctly, it will look like this [show sample]. A line from 3 to 5 to 6 would be incorrect [pointing], as the proper order is 3, 6, 5. How many have it drawn exactly like this [pointing to sample]? Raise your hands [assistant to discover if anyone has it wrong and to explain the procedure to him individually].

Let us try one more. Watch carefully [expose Card No. 8 for six seconds and then show the reverse]. Find Square No. 8 and draw the proper lines [allow ten seconds]. If you have done it correctly, it will look like this [show sample]. How many have it exactly like this? Raise your hands [assistant to discover if there still are errors. If there are, give Card No. 4 as a practice card. If there are no errors, skip this and start with the test proper, Card No. 4, as follows]. Now, I am going to show some more cards, but from now on I am not going to tell you whether you are right or not, so look at the cards carefully [show cards in order, exposing the front, the number of seconds indicated in small print in the corner on the front, and allow writing time one and one-half times as long. Precede each flash by, "The next card." When the bottom of the page is reached, No. 24, state] If you forget part of the words, numbers, or symbols shown, do not be disturbed, but do the best you can and draw lines connecting as many as you remember. Now, we will start on the next page. The next card is this one [continue as before to the end of the test, allowing a one-minute break at the end of the second page].

The print on the flash cards was in lower-case eight-pica type, and symbols were approximately eight picas in height. In Table XXXVI following, symbols are designated by numbers, No. 1 being at the top of the designated square and the numbering thence 2, 3 . . . . , 8 in a clockwise direction.

TABLE XXXVI

ORDER OF EXPOSURE OF CARDS USED IN MEMORY TESTS

Order of Presentation	Symbols, Words, etc.	Exposure Time in Seconds	Square in Which Drawing Is to Be Done
Form A			
1	of, to, on [correctly marked samples shown]	10	2
2	3, 6, 5 [correctly marked samples shown]	10	11
3	No. 8, No. 4 [correctly marked samples shown]	8	8
4	is, so, my, at	6	4
5	he, an, in	5	1
6	No. 1, No. 7	6	16
7	No. 5, No. 7	4	7
8	if, am, or	5	3
9	2, 9, 4, 3	6	13
10	do, be, no, us	6	5
11	No. 2, No. 5	6	17

TABLE XXXVI—*Continued*

Order of Presentation	Symbols, Words, etc.	Exposure Time in Seconds	Square in Which Drawing Is to Be Done
12	No. 7, No. 2	4	6
13	no, at, to, if, do, we	8	23
14	5, 2, 6, 8	6	14
15	my, go, of, as, if	7	21
16	No. 4, No. 8	6	19
17	6, 4, 2	5	12
18	No. 3, No. 1	4	9
19	am, we, to, us, it	7	22
20	No. 5, No. 7	6	18
21	No. 2, No. 6	4	10
22	3, 2, 4, 9, 7	7	15
23	No. 2, No. 8	6	20
24	it, up, no, we, be, am	8	24
	[Turn over to page 2]		
25	67, 96	5	30
26	No. 2, No. 8, No. 5	5	26
27	No. 7, No. 5	6	36
28	way, but, own	5	40
29	No. 8, No. 6, No. 3	5	29
30	73, 97, 85	6	32
31	No. 1, No. 4	4	25
32	No. 3, No. 8	6	35
33	17, 56, 93, 31	8	33
34	No. 7, No. 1, No. 5	5	28
35	got, did, him, out, she	7	42
36	No. 7, No. 3, No. 1	5	27
37	89, 46, 27, 59	8	34
38	No. 7, No. 4	6	37
39	has, get, she, air	6	41
40	No. 7, No. 1, No. 3	5	44
41	No. 2, No. 7, No. 4	8	39
42	No. 3, No. 7, No. 2	5	46
43	49, 74, 11	6	31
44	No. 5, No. 3, No. 8	5	48
45	No. 3, No. 8, No. 5	5	45
46	No. 6, No. 1, No. 5	8	38
47	its, pay, any, not, day, the	8	43
48	No. 1, No. 4, No. 6	5	47
49	No. 8, No. 2, No. 7	8	55

TABLE XXXVI—*Continued*

Order of Presentation	Symbols, Words, etc.	Exposure Time in Seconds	Square in Which Drawing Is to Be Done
50	264, 979, 622	8	51
51	rich, pass, wait, each, were	7	61
52	No. 1, No. 7, No. 4	8	57
53	692, 787	6	49
54	982, 443, 685, 630	10	53
55	No. 4, No. 8, No. 6	8	54
56	669, 383	6	50
57	tell, does, live	5	59
58	No. 1, No. 4, No. 6	5	63
59	late, hope, wish, paid	6	60
60	No. 3, No. 5, No. 1	5	65
61	365, 539, 307	8	52
62	No. 2, No. 4, No. 1	5	66
63	No. 1, No. 6, No. 4	8	69
64	No. 5, No. 3, No. 1	8	56
65	No. 2, No. 6, No. 3	8	72
66	No. 6, No. 3, No. 7	8	70
67	will, done, move, lost, fine, want	8	62
68	No. 8, No. 6, No. 3	5	64
69	No. 8, No. 6, No. 1	8	71
70	No. 1, No. 4, No. 8	8	58
71	No. 8, No. 4, No. 7	5	67
72	No. 3, No. 5, No. 8	8	68
	[Turn over to page 4]		
73	other, along, begin	5	73
74	No. 3, No. 6, No. 2, No. 4	6	78
75	No. 6, No. 8, No. 5	8	82
76	No. 2, No. 4, No. 1, No. 6	5	89
77	No. 3, No. 7, No. 2, No. 6	7	95
78	happy, often, about, visit, being	7	75
79	No. 8, No. 3, No. 5	8	84
80	No. 8, No. 2, No. 6, No. 4	6	80
81	No. 3, No. 5, No. 2, No. 6	5	88
82	No. 2, No. 5, No. 8, No. 4	8	86
83	No. 6, No. 3, No. 5	8	85
84	No. 1, No. 5, No. 2, No. 7	6	77
85	No. 6, No. 1, No. 3, No. 8	7	93
86	visit, about, heavy, never	6	74
87	No. 7, No. 4, No. 2, No. 6	6	81

TABLE XXXVI—*Continued*

Order of Presentation	Symbols, Words, etc.	Exposure Time in Seconds	Square in Which Drawing Is to Be Done
88	No. 7, No. 5, No. 3, No. 8, No. 4	6	91
89	No. 3, No. 6, No. 2, No. 8	7	94
90	No. 3, No. 5, No. 1, No. 4, No. 8	6	90
91	No. 3, No. 1, No. 4, No. 2	7	96
92	No. 2, No. 6, No. 1, No. 3	5	87
93	until, heavy, never, about, cover, today	8	76
94	No. 1, No. 4, No. 8	8	83
95	No. 3, No. 5, No. 2, No. 7	6	79
96	No. 8, No. 3, No. 7, No. 5	7	92
Form B			
1	as, we, on [correctly marked samples shown]	10	2
2	7, 9, 6 [correctly marked samples shown]	10	11
3	No. 4, No. 8 [correctly marked samples shown]	8	8
4	as, is, go, at	6	4
5	do, it, be	5	1
6	No. 6, No. 2	6	16
7	No. 3, No. 6	4	7
8	he, up, am	5	3
9	8, 6, 9, 4	6	13
10	us, or, am, at	6	5
11	No. 5, No. 7	6	17
12	No. 8, No. 6	4	6
13	it, to, an, so, be, if	8	23
14	9, 3, 6, 7	6	14
15	if, up, my, go, no	7	21
16	No. 1, No. 7	6	19
17	6, 5, 9	5	12
18	No. 5, No. 1	4	9
19	so, on, is, or, us	7	22
20	No. 2, No. 6	6	18
21	No. 4, No. 7	4	10
22	6, 5, 8, 2, 4	7	15
23	No. 8, No. 3	6	20
24	so, up, of, he, as, on	8	24
	[Turn over to page 2]		
25	39, 82	5	30
26	No. 7, No. 3, No. 8	5	26
27	No. 4, No. 8	6	36

TABLE XXXVI—*Continued*

Order of Presentation	Symbols, Words, etc.	Exposure Time in Seconds	Square in Which Drawing Is to Be Done
28	off, our, ask	5	40
29	No. 8, No. 2, No. 7	5	29
30	55, 86, 79	6	32
31	No. 6, No. 2	4	25
32	No. 2, No. 7	6	35
33	13, 94, 21, 50	8	33
34	No. 2, No. 7, No. 4	5	28
35	law, sit, yet, too, and	7	42
36	No. 2, No. 8, No. 6	5	27
37	95, 26, 88, 71	8	34
38	No. 3, No. 5	6	37
39	son, let, old, out	6	41
40	No. 4, No. 8, No. 3	5	44
41	No. 2, No. 4, No. 6	8	39
42	No. 2, No. 4, No. 7	5	46
43	12, 32, 64	6	31
44	No. 3, No. 5, No. 7	5	48
45	No. 5, No. 8, No. 2	5	45
46	No. 2, No. 7, No. 5	8	38
47	hot, may, ask, new, for, two	8	43
48	No. 4, No. 2, No. 7	5	47
49	No. 5, No. 1, No. 3	8	55
50	140, 958, 249	8	51
51	care, sure, made, hold, keep	7	61
52	No. 8, No. 4, No. 7	8	57
53	502, 764	6	49
54	818, 873, 735, 521	10	53
55	No. 8, No. 6, No. 1	8	54
56	455, 288	6	50
57	read, call, mean	5	59
58	No. 6, No. 3, No. 7	5	63
59	came, best, draw, soft	6	60
60	No. 8, No. 2, No. 4	5	65
61	388, 794, 945	8	52
62	No. 8, No. 2, No. 6	5	66
63	No. 8, No. 2, No. 7	8	69
64	No. 1, No. 7, No. 4	8	56
65	No. 7, No. 5, No. 2	8	72
66	No. 2, No. 7, No. 4	8	70

TABLE XXXVI—*Concluded*

Order of Presentation	Symbols, Words, etc.	Exposure Time in Seconds	Square in Which Drawing Is to Be Done
67	love, take, have, then, help, true	8	62
68	No. 2, No. 6, No. 4	5	64
69	No. 7, No. 5, No. 8	8	71
70	No. 2, No. 6, No. 4	8	58
71	No. 4, No. 7, No. 1	5	67
72	No. 3, No. 7, No. 5	8	68
	[Turn over to page 4]		
73	early, again, money	5	73
74	No. 6, No. 3, No. 1, No. 4	6	78
75	No. 2, No. 6, No. 3	8	82
76	No. 7, No. 1, No. 6, No. 4	5	89
77	No. 8, No. 2, No. 7, No. 1	7	95
78	carry, given, today, order, never	7	75
79	No. 3, No. 7, No. 4	8	84
80	No. 6, No. 4, No. 7, No. 5	6	80
81	No. 4, No. 1, No. 3, No. 6	5	88
82	No. 3, No. 7, No. 4, No. 8	8	86
83	No. 2, No. 4, No. 8	8	85
84	No. 2, No. 6, No. 8, No. 4	6	77
85	No. 5, No. 7, No. 4, No. 8	7	93
86	today, alone, often, until	6	74
87	No. 1, No. 6, No. 4, No. 7	6	81
88	No. 6, No. 3, No. 1, No. 4, No. 2	6	91
89	No. 7, No. 3, No. 8, No. 2	7	94
90	No. 7, No. 1, No. 6, No. 4, No. 8	6	90
91	No. 7, No. 1, No. 5, No. 3	7	96
92	No. 1, No. 6, No. 3, No. 8	5	87
93	visit, story, being, money, often, after	8	76
94	No. 7, No. 2, No. 4	8	83
95	No. 3, No. 7, No. 5, No. 2	6	79
96	No. 6, No. 2, No. 7, No. 4	7	92

Part of the fourth page dealing with meaningless and meaningful symbols proved too difficult for pupils of both grades studied. The same administrative procedure was followed in the third grade as in the seventh, except that none of the difficult fourth page of the test was employed.

For each part, words, numbers, meaningful symbols, and meaningless symbols, various scoring schemes were investigated, for the population of 140, with a view to determining the optimum procedure. No separate scoring scheme study was made for these four tests, or for Tests 9 and 10, for the third-grade group. The scoring devices actually adopted are the same for both the third and seventh grades, and differ slightly from the statistically determined optima for the seventh-grade population. If S equals the number of squares (word squares in the verbal memory test, number squares in the number memory test, etc.) correctly marked in every respect, and L equals the number of lines correctly drawn (i.e., if two only out of three lines for a given square are correctly drawn, a credit of two is given, etc.) the general scoring scheme investigated may be represented by $(S + cL)$. In all cases c proved either so small or so large that it was not necessary to use both S and L in the final scoring device. The scores actually used are given for each of the four tests in the following paragraphs:

Memory for Words.—The score is L, or the number of correctly drawn lines. This score has a reliability for the 140 population of .802 (the optimum correlation based upon $S - 4.9\,L$ was .803, which is negligibly higher than .802), for the 109 population of .756, for the 110 population of .740, and for the 60 population of .732.

Memory for Numbers.—The score is S, or the number of squares marked correctly in every respect. This has a reliability for the 140 population of .457 (the optimum is .462), for the 109 population of .444, for the 110 population of .556, and for the 60 population of .537.

Memory for Meaningful Symbols.—The score is L, or the number of correctly drawn lines. This has a reliability for the 140 population of .7417 (the optimum is .7423), for the 109 population of .729, for the 110 population of .542, and for the 60 population of .404.

Memory for Meaningless Symbols.—The score is L, or the number of correctly drawn lines. This has a reliability for the 140 population of .708 (the optimum is .712), for the 109 population of .731, for the 110 population of .583, for the 60 population of .493.

Manipulation of Spatial Relationships Speed Test (Test 10).— This test was given before Test 9, the spatial power test. In fact, it was due to the difficulties discovered in the administration of Test 10, and largely overcome in giving Test 9, that fair scores were secured throughout on Test 9. Many of the pupils did not adequately grasp the directions for Test 10, with the result that both the third- and seventh-grade populations, for which we have fair Test 10 scores, are considerably reduced in size. For certain seventh-grade classes the only directions were those printed on the blank (see page 173). These proved inadequate and were later supplemented both in the seventh grade and in the third grade, the following statement being representative of the third-grade administration (page 173 is before the pupils).

Look at the first row of figures for the "figure exercises." I have here a chart on the blackboard just like the one that is drawn on the papers. Here is the row of figures *A*, *B*, 1, 2, 3, 4, 5, 6, and 7. Each of the figures to the right of this heavy line can be cut up into the two figures *A* and *B*. With your pencil draw a line in each of these figures 1, 2, 3, 4, 5, 6, and 7, cutting it up into two figures like *A* and *B*. Do it right now. [After one and one-half minutes] Stop. [Draw and point to the blackboard figures as necessary.] Now, if you drew the line correctly in this first figure you will have a line right down here, because that cuts this one off, which is the same as this. In Figure 3 you would have a line right across here. Is there anyone who doesn't understand what I want you to do? Now, look at this upper part. Can this upper part be turned around without turning over so that it will fit Figure *A*? [Examiner demonstrates with the aid of cardboard samples of figures.] No, it cannot, so I want you to write a letter *t* in the upper part. The *t* means that the upper part has to be "turned over" to fit. Now, let us look at Figure 4. Does this need to be turned over? No, it does not. Nothing needs to be turned over here. Look at Figure 5. This part does not need to be turned over, neither does this other part. Look at Figure 6. Does the upper part need to be turned over? Yes, it has to be turned over; I cannot make it fit otherwise. So I want you to write the letter *t* in the upper part. Look at Figure 7. Does anything need to be turned over here? No, so you do not need to write *t*. Now, let us look at the row at the bottom of the page. You want to remember that every figure 1, 2, 3, 4, 5, 6, 7 is to be divided by a line cutting it into two parts, thus making figures *A* and *B*, and second, you are to write the letter *t* in each part that must be turned over in order to make Figure *A*. Do this whole row on your paper as rapidly as you can. Do it right now. [After one and one-half minutes] Stop. Now, on pages 6 and 7 we have a lot more just like this. I want you to do the same thing. Be sure to have each one right, but work as fast as you can. Work with your blue pencils. Ready, go. [After 11 minutes in the third grade and 8 minutes in the seventh grade] Stop. [This terminated the test for the third grade, but for the seventh it continued as follows:] Use your red pencils and continue. [After 3 minutes. At one sitting for one class, but 2 minutes was given here and an adjustment in the score was made to allow for this shortened time.] Stop. Will the children in the back seats collect the papers.

For the seventh grade, scores for both the eight- and the eleven-minute periods were investigated, but the final score is based upon a working time of eight minutes for both third and seventh grades. Three scores were studied, (*a*) S_1, the number attempted, (*b*) S_2, the number of figures correctly "lined" and correctly "*t*'d," i.e., the number that had been correctly divided into parts, and also in which *t*'s occurred in just those parts that require turning over, (*c*) S_3, which is given by the equation

$$S_3 = S_2 + \tfrac{1}{4} \text{ (the number correctly lined but incorrectly } t\text{'d).}$$

Optimum scores involving $(S_2 + cS_3)$, $(S_2 + cS_1)$, and $(S_3 + cS_1)$ were determined. The score finally decided upon for the seventh grade was S_2, or the number correctly lined and correctly *t*'d in eight minutes. This has a reliability for the population of 109 of .738 (the optimum reliability was .753).

For the third grade the scoring scheme adopted for the seventh grade was used without a redetermination of the optimum scheme, but the working time was eleven minutes instead of eight. For the population of 60 the reliability is .575.

Manipulation of Spatial Relationships Power Test (*Test 9*).—The directions as printed on the test blank (see page 178) were first carefully followed. As these proved insufficient, additional practice, using a large-scale sheet, showing row *a* of the test proper and the figures at the top, was pinned to the wall and utilized as follows:

In Row *a* (Form A), just below the heavy line at the top of the page, there are some figures made up of the figure at the left of the heavy line and one figure of those numbered at the top 1, 2, 3, 4, 5, 6, 7, 8. Which figure at the top is required? Figure 6 is correct, so write the number "6" at the beginning of the row right next to the little letter *a*. Now, do this row just as you did the sample row on the directions page. Write above each figure the number of parts that must be turned over. Start now, but do not continue with the second row until I tell you to. [If, due to transfer from Test 10, certain pupils write *t* on the figures, they should be individually advised as to the correct procedure. Allow two minutes in the third grade and one minute in the seventh.] Stop.

Look at Row *b*. What figure at the top must be used in connection with the figure at the left of the heavy line to make the figures of this row? Yes, Figure 1 at the top is required. This figure is also required for Row *c*, so write the number "1" beside the small letters *b* and *c*, and then do these two rows. Ready, go. [Allow 3 minutes in third grade and 2 minutes in seventh grade.] Stop.

Now look at Row *d*. What figure at the top is needed? Figure 8 is required, so write the number "8" beside the small letter *d*, and do this row as you did the first three. In this row each figure is made up of three separate parts instead of two, so you will need to draw two lines cutting up each figure, and you must write down the number showing the number of pieces that must be turned over. Remember, never turn a piece over unless you are unable to make it fit in any other way. [Allow 2 minutes in third grade and 1 minute 15 seconds in seventh.] Stop.

Look at Row *e*. Figure 4 at the top is needed, so write the number "4" next to letter *e* and do this row. [Allow 2 minutes 20 seconds in third grade and 1 minute 50 seconds in seventh.] Stop.

Look at Row *f*. Figure 7 at the top is needed, both for this row and for Row *g*, so write the number "7" beside letters *f* and *g* and do these two rows. [Allow 4 minutes 30 seconds in third grade and 3 minutes 40 seconds in seventh. As this is too hard for most third-grade pupils, assistant may tell pupils singly, who have plainly reached their limit, not to attempt any more.] Stop.

Look at Row *h*. Figure 2 at the top is needed, so write the number "2" next to the letter *h* and do this row, but *do not go on to Rows i and j* [assistant is to see that this instruction is obeyed]. Remember you are not to draw any lines in Rows *i* and *j* until I tell you to. Ready, go. [Allow 3 minutes in third grade and 3 minutes 40 seconds in seventh.] Stop.

Now look at Row *i*. Figure 2 is needed for this row, and also for Row *j*, so write "2" next to letters *i* and *j*. In these two rows I want you to write down as before the number of parts necessary to turn over, but here you must *not draw any lines* cutting up the various figures. Just imagine these lines; do not draw them. I know it is harder to determine how many parts have to be turned over when no lines cutting up the figures are drawn, but I want to see if you are able to imagine the lines without drawing them, so write above every figure in Rows *i* and *j* the number of parts that must be turned over, but do not draw any lines cutting up these figures. [Assistant must see that this instruction is rigidly followed. If any pupil shows an inclination to draw light lines or draw lines and then erase them, he must be kept under surveillance so that this is impossible.] Ready, go. [Not given to third grade. Allow 6 minutes in the seventh.] Stop.

The scores investigated were P_1, the number of figures correctly lined and numbered as regards the number of parts necessary to turn over, and P_2, which is equal to $(P_1 + \frac{1}{4}P_3)$, in which P_3 is the number correctly lined but incorrectly numbered. Optimum scoring schemes were found for both third and seventh grades, but that finally used differed somewhat from both of these. It was $(P_1 + \frac{1}{19} P_2)$. Though this is the form in which it was actually used, it is readily seen that this is equivalent to $(P_1 + \frac{1}{80} P_3)$, or the number of figures correct in every respect, plus $\frac{1}{80}$ of the number correctly lined but incorrectly numbered. With reference to further work the writer would say that there is not sufficient difference between this scoring scheme and the use of P_1 alone to warrant its use. The scoring scheme has a reliability for the 140 population of .781 (the optimum would have been .782), for the 109 population of .753, for the 110 population of .710, and for the 60 population of .612.

Copies of all the new tests used, i.e., of Numbers 1, 3, 5, 6, 7, 8, 9, 10 are given herewith, reduced to just ⅗ the original size, together with such printed instructions as appeared on the test blanks. The instructions were not repeated in full when the second forms of the tests were given.

FORM A

BOOKLET OF SPEED TESTS (PAGE 1)

DIRECTIONS

Name .... **Boy or Girl**........................ **Date**........................
(First name) (Last name)

Age............ **When is your next birthday?**.. **School**........................

How old will you be then?.. **Grade**........................

READING EXERCISE

In the following story there is a parenthesis every few words. In each parenthesis are two words, one of which does not belong there because it does not make good sense in the sentence. You are to read these sentences when I tell you to start and whenever you come to a parenthesis, rapidly draw a line under the right word, and then continue reading as fast as you can, until I tell you to stop. Ready—go.

WHEN THE TRAIN PASSES

I live within one-half mile of the railroad (map, tracks). I see the trains pass many times and it always gives me a (thrill, penny) to watch them. First is the big, black engine in (front, school), making an awful noise, spitting fire and racing down the shiny (lake, rails). I like to watch it from beneath a big oak (chair, tree) by the roadside. With such a monster loose, one needs a friendly oak to (protect, scare) him.

Stop (after 15 seconds). What word did you draw a line under in the first parenthesis? "Tracks" is right, because now the sentence reads, "I live within one-half mile of the railroad tracks." What word did you draw a line under in the next parenthesis? "Thrill" is right; and in the next parenthesis the right word is "front." On Pages 2 and 3 are two other exercises in which I want you to do the same thing. In each parenthesis draw a line under the right word. Do it as fast as you can. Turn to Page 2. Ready—go.
Stop (after just five minutes).

READING EXERCISE

FUN IN THE SNOW

When I went to bed that (morning, night), it was very windy and cold. There were few clouds in the (sky, road), so I was quite surprised the next (time, morning) to hear Father say, "Get up, Harry, and see the (snow, sunshine). It has been snowing all night, and now everything is (green, white). Come to the window and see." And Mother said, "I am afraid there is so much (snow, dust) on the ground that Harry will not be able to go to school today." Did I bury my head in the (pillow, sand) and cry over this sad news? No! I did not. I ran to the window and looked (in, out). Sure enough! The whole world outside was covered with white snow, piled high about the (sides, tables) of the houses and the walls, where it had been blown by the (fan, wind). In our back yard there were deep holes (in, above) the snow made by the feet of the milkman, who comes early in the (springtime, morning) with the milk.

After breakfast the sun came out and (spoke, shone) brightly. The snow began to melt a little. This made it very good for making (toys, snowballs). When I looked out the back window I saw that Mr. Thomas, the (man, woman) who lives next door, was digging a path from (his, her) house to ours. Soon Billy Thomas came out and (whispered, shouted) to me to come and help. Billy is half a year older than I, but we (live, play) together all the time.

Mother gave me my brown coat and wool cap, and I put on my big boots and (looked, ran) out. The snow was up to my knees in most places. I began to help Mr. Thomas (dig, measure) the path, but when Billy made a snowball and threw it at me, (I, she) found it more interesting to make snowballs than to dig the path. That started a (rock, snow) fight, but it was only in fun. The soft snow did not hurt at all. Billy's two big (brothers, sisters), Jenny and Alice, came out of the house and we both attacked them fiercely with the biggest (stones, snowballs) we could throw. Jenny caught me and (Alice, Gertrude) caught Billy and they washed our faces in the snow. We decided it was not polite to throw snowballs (at, for) girls, so we stopped and asked them if they wouldn't help us make a (cart, snow) man. We rolled a great big snowball for the man's body and a smaller one for his (coat, head). In this we put pieces of coal for eyes. His nose was an old stick of wood, and we (put, found) an old hat on his head. Billy called the man Mr. Williams,—that's Jenny's beau,—and he got his (hands, face) washed a second time.

During the next night it became very (sunny, cold) and the softened snow had frozen so hard that the surface would bear our (weight, boat) and we walked over the top of the snow instead of wading (through, around) it. Billy Thomas and I began to dig the (soft, black) snow from beneath the crust and soon had a lovely cave under a snowbank beside the (path, bed). Although it was very cold outside, it was quite warm in the (shed, cave) and we could look up through our (icy, shingle) roof at the sun and found it was like (looking, walking) through frosted glass, only the glass seemed to be made of rainbows.

As the weather continued very (hot, cold), we poured a little water over our roof each night and soon had a very strong coating of (ice, paint) which lasted several weeks. We worked each (day, year) making the cave a little larger and finally had two (rooms, boxes) with walls of snow between. We could stand upright in the (center, edge) of the cave, but Billy and I are not very (heavy, tall).

The snow man stood guard (beside, inside) the cave and we made everyone who came to see us leave a (rubber, snow) ball at his feet. Soon we had a large store of (chestnuts, ammunition), and then we had a party and a big (pillow, snowball) fight. Anyone who could break through the line of (defense, march) and get into the cave, had the right to fire these (rubber, snow) balls from the rear. We were very hungry when the fight finished and Mother let us have hot doughnuts to (eat, sell) in the cave.

Go right on to the next story and do the same thing.

FORM A

Booklet of Speed Tests (Page 3)

JACK'S KITE

"Look at what my papa (sang, gave) me. Isn't it a beauty? I bet it will fly higher than any other (kite, cart) in town." Jack's kite was certainly a (bitter, nice) one. It had a dragon's head right on the (front, inside) of it. Its tail was made of little bunches of paper tied on a (pole, string), and each bunch was of a different (frame, color). All the children wanted to see it fly, so they went (up, tonight) to the top of the hill, where there was more (wind, water). Jack didn't know very much about flying kites and Arthur offered to (show, ask) him how. Jack was afraid to let anybody else (make, touch) his fine kite. He would not even let Arthur hold it while he pulled the (string, plow). Instead of letting anyone help him get it into the air he tried to throw it up and get it (started, back) that way. He did not succeed and the kite fell down with a bang. Jack was afraid that it was broken. When he picked it up the dragon looked as fierce as ever and the tail danced in the (wind, winter). Everything seemed to be all right. He tried again, but this time he let Arthur help (him, us). Jack let out quite a bit of string and Arthur held the (clock, kite), high above his head, ready to let it go when Jack should pull the (string, grain). Just then a good breeze sprang up and Jack called (Joe, out), "Let her go!" Immediately the kite soared upward and soon it was way up by the chimney tops. The red and green dragon was bobbing about and wagging its tail in great (size, style). Jack felt so happy that he asked Arthur if he didn't want to (stop, fly) it for a while

Arthur wrote on a small piece of paper, "Hello! Blue Sky, isn't this a fine (box, kite)?" He made a little hole in the paper and put it on the string and the (fan, wind) blew it away up to the kite. Almost as soon as the message got up there a strong puff of (wind, smoke) struck the kite and made it dance vigorously. Arthur cried out in glee, "That is Blue Sky saying that he thinks it is a mighty fine (cart, kite)." Another puff tore the message from the (boy, string) and carried it higher and higher until it was finally lost to sight. Arthur said, "I suppose (Clear, Blue) Sky thought it would not be polite to send our message back, so he just took it away (with, from) him." Blue Sky developed very decided opinions of his own and continually made the kite race back and (in, forth) and back and forth and dive and (soar, swim). Jack was afraid the entire kite would break (loose, somebody) and blow away, so he asked Arthur to pull it (out, in). This seemed to make Blue Sky mad, for the kite jumped and pulled harder than (rocks, ever) and finally in a grand swing dove down to the ground (back, inside) of a house. Jack began to cry, thinking his kite was (gathered, broken) into pieces. They went to find it and soon saw its tail way up on a (piano, telephone) wire. Soon they found the kite itself sticking in the mud of a vegetable (cart, garden), none the worse for its wild flight except that it had lost its (tail, mind). Jack didn't know whether to cry or not, but Arthur said, "Come on, let's (go, fly) home" and he sang out, "It hung its tail on a telephone (wire, pole) and it sat down nicely in the mire." Jack's tears disappeared and as (we, they) trudged home he joined in the chorus, "And it sat down nicely in the (chair, mire)."

ARITHMETIC EXERCISE

Add:

				5	4	1	8
8	9	2	8	6	9	9	5
3	7	7	6	8	7	7	4
13, 11	16, 14	9, 7	12, 14	22, 19	20, 23	17, 14	13, 17

Add:

4	5	3	9	5	5	8	2
3	7	7	4	9	4	4	6
5	9	7	6	8	7	7	8
12, 15	19, 21	19, 17	19, 16	22, 26	16, 17	19, 16	16, 18

Subtract:

11	16	18	12	15	19	19	11
7	5	3	7	6	8	4	6
3, 4	9, 11	17, 15	5, 9	9, 6	13, 11	15, 13	3, 5

Subtract:

14	18	17	15	12	13	16	19
9	4	5	7	8	6	7	7
5, 7	14, 16	11, 12	8, 5	4, 5	6, 7	7, 9	12, 14

Multiply:

$2 \times 5 = 15, 10$ | $5 \times 3 = 15, 20$ | $3 \times 2 = 6, 8$ | $4 \times 3 = 14, 12$

Multiply:

$4 \times 5 = 25, 20$ | $5 \times 5 = 25, 35$ | $4 \times 4 = 16, 18$ | $2 \times 4 = 8, 12$

Divide:

$10 \div 2 = 4, 5$ | $8 \div 2 = 4, 2$ | $8 \div 4 = 2, 3$ | $6 \div 2 = 2, 3$

Divide:

$6 \div 3 = 2, 3$ | $10 \div 5 = 3, 2$ | $4 \div 2 = 3, 2$ | $9 \div 3 = 3, 4$

Add:

9	2	5	9	8	6	3	9
6	5	2	3	2	7	9	2
8	2	7	8	8	4	8	7
3	7	4	5	7	3	5	8
26, 19	13, 16	17, 18	15, 25	25, 23	20, 24	25, 27	26, 28

Go right on with the problems on page 5.

FORM A

Booklet of Speed Tests (Page 5)

Add:

6	3	8	2	9	7	8	3
7	5	5	9	9	8	9	6
9	9	6	4	8	3	7	1
9	6	7	6	7	9	8	8
33, 31	23, 17	26, 25	21, 23	32, 33	31, 27	35, 32	18, 16

Subtract:

23	22	28	28	20	27	25	21
17	13	15	19	13	16	17	14
6, 5	9, 7	13, 16	9, 7	5, 7	11, 7	4, 8	8, 7

Subtract:

29	29	27	26	29	29	26	28
14	12	13	18	11	17	19	16
11, 15	17, 15	13, 14	6, 8	16, 18	12, 14	8, 7	17, 12

Multiply:

$4 \times 8 = 32, 36$ $5 \times 7 = 35, 36$ $5 \times 6 = 30, 35$ $4 \times 9 = 38, 36$

Multiply:

$4 \times 7 = 27, 28$ $2 \times 7 = 17, 14$ $4 \times 8 = 38, 32$ $5 \times 9 = 49, 45$

Divide:

$14 \div 2 = 7, 6$ $15 \div 3 = 5, 10$ $18 \div 3 = 8, 6$ $12 \div 4 = 3, 4$

Divide:

$20 \div 4 = 6, 5$ $16 \div 4 = 4, 3$ $18 \div 6 = 4, 3$ $18 \div 9 = 4, 2$

Multiply:

$6 \times 9 = 54, 57$ $8 \times 7 = 48, 56$ $7 \times 6 = 42, 46$ $9 \times 8 = 68, 72$

Multiply:

$9 \times 7 = 63, 72$ $7 \times 7 = 46, 49$ $9 \times 9 = 89, 81$ $6 \times 8 = 52, 48$

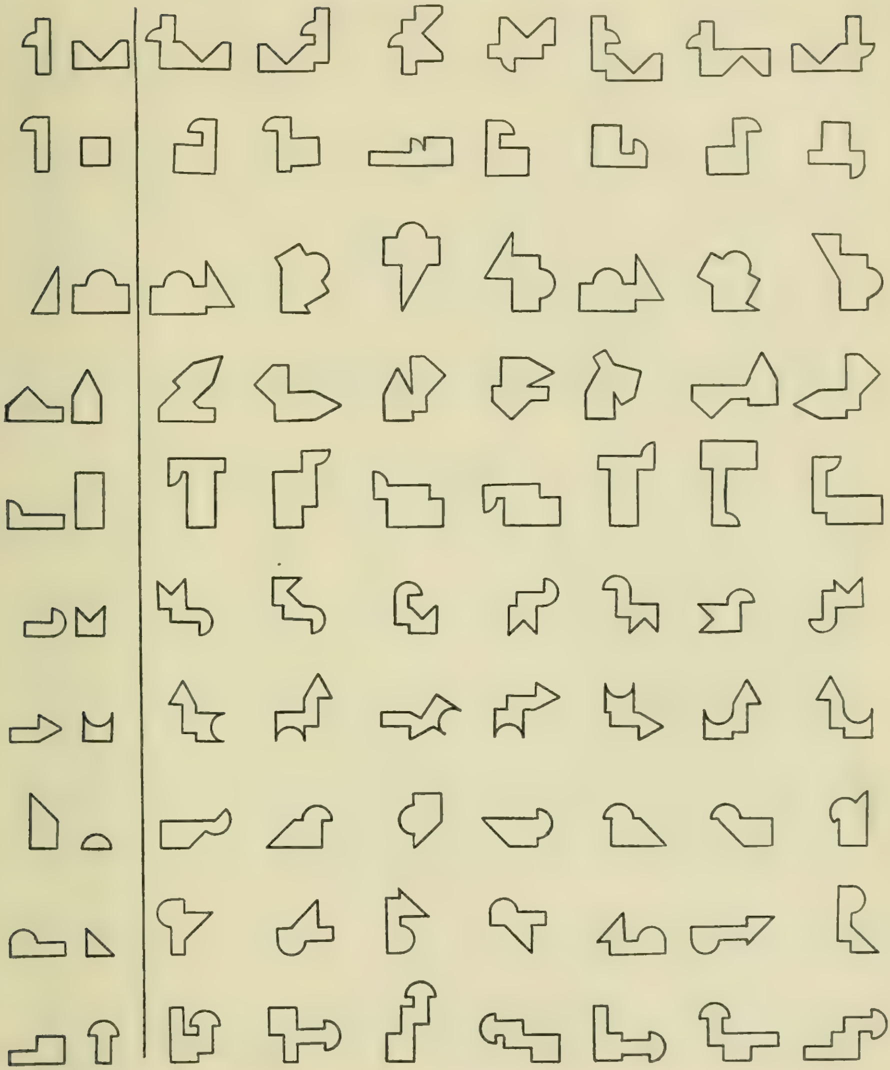
Divide:

$50 \div 10 = 5, 10$ $36 \div 12 = 3, 4$ $44 \div 11 = 6, 4$ $48 \div 12 = 4, 6$

Divide:

$38 \div 19 = 2, 3$ $45 \div 15 = 3, 2$ $39 \div 13 = 3, 6$ $42 \div 14 = 5, 3$

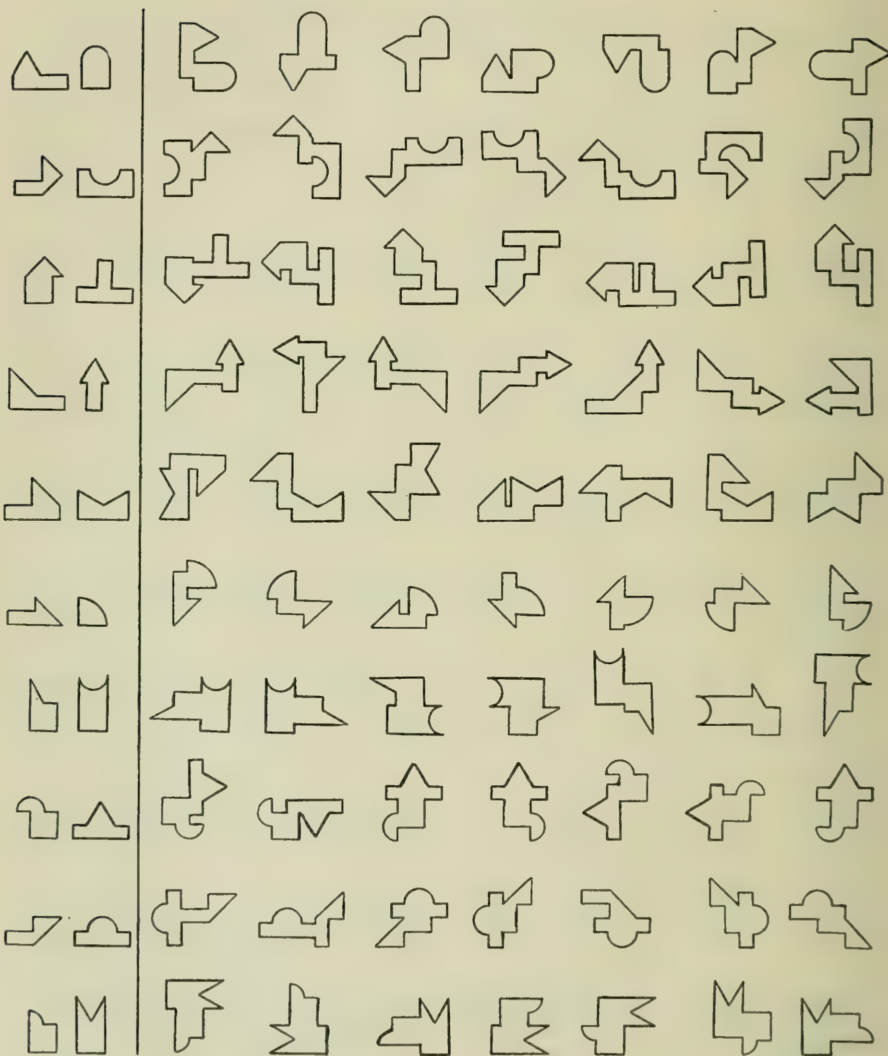
FIGURE EXERCISE No. 1



Go right on to the next page.

FORM A

Booklet of Speed Tests (Page 7)



COMPUTATION EXERCISES

Below are some arithmetic problems. After each problem are two answers, but only one of them is right. You are to do the problems in order as you come to them, drawing a line under the correct answer. Ready—go.

Add:			Multiply:
	2	3	$3 \times 4 = 22$, 12
	3	4	
	6	2	Divide:
	11, 13	7, 9	$15 \div 5 = 5$, 3

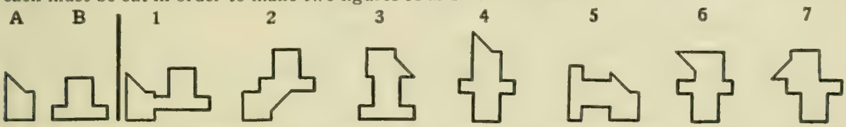
(Allow ½ minute.)

The number that should have a line under it in the first problem is 11. In the second problem it is nine. In the third problem there should be a line under 12, and in the fourth problem under 3. How many got them all right?

Now turn to pages 4 and 5 and work the problems in order. Work as fast as you can. Draw a line under the correct answer. Ready—go. (Allow 3 minutes.) Stop.

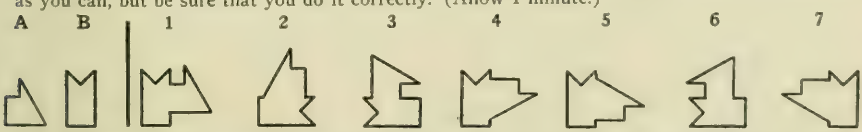
DIRECTIONS FOR FIGURE EXERCISE No. 1

Below is a row of figures, A, B, 1, 2, 3, 4, 5, 6, 7. Each of the figures 1, 2, 3, 4, 5, 6, 7 to the right of the heavy line can be cut up into the two figures A and B shown at the left of the line. With your pencil draw a line on each of the figures numbered 1, 2, 3, 4, 5, 6, 7 showing just how each must be cut in order to make two figures A and B. Do it now.



(Allow ½ minute.) Look closely at figure 3. If you cut it where you have just drawn a line, can you turn the upper part around without lifting it from a flat table top so as to make figure A, or is it necessary that you turn this upper part over in order to make figure A? It must be turned over, for otherwise it is impossible to make it fit exactly on figure A. Therefore I want you to write the small letter "t" meaning "turned over" on this upper part of figure 3. Let us see if we can find another figure in which it is necessary to turn the piece over before it will exactly match figure A. How about figure 1? No part of figure 1 needs to be turned over to make figure A, so we will go on to figure 2. Look carefully. Can the part cut off be moved around on a flat table top without turning it over and be made to fit exactly on figure A? Yes, it can, so we will go on to figure 4. Here again nothing needs to be turned over, so we will look at figure 5. Again nothing needs to be turned over, so we will look at figure 6. There is no possible way of making the part cut off fit figure A, by moving it around on a table top. It simply must be turned over. Therefore I want you to write the letter "t" in the upper part of figure 6. That is, in the part which must be turned over in order to make figure A. Look at figure 7. Nothing needs to be turned over.

Below is another row in which you are to do the same thing. Remember in each figure 1, 2, 3, 4, 5, 6, 7, draw a line cutting it into two parts making figures A and B, and second, write the letter "t" in each part that must be turned over in order to make figure A. Do the row as rapidly as you can, but be sure that you do it correctly. (Allow 1 minute.)



On Pages 6 and 7 are more rows of figures on which I want you to do the same thing. Turn to Page 6. Be sure you have each one right, but work as fast as you can. Ready—go. (Allow 11 minutes.)

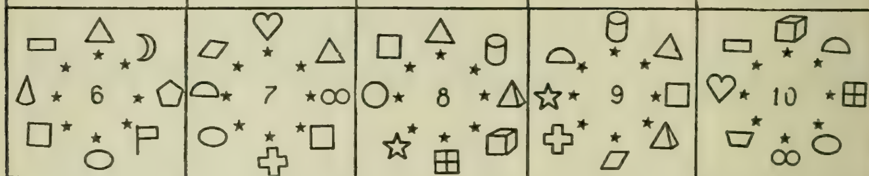
FORM A

Memory Tests (Page 1)

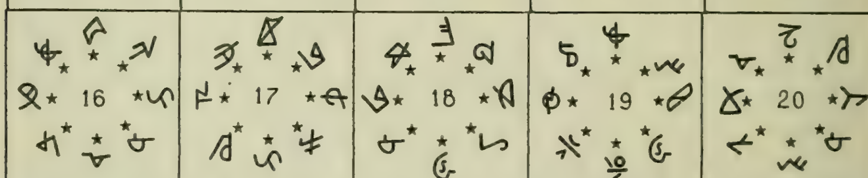
CIRCLES TEST

Name..
First name Last name

1	an	in, to, am	be, us, up	he
2	is	of, as, it	to, he, at	on
3	go	am, is, do	we, no, or	if
4	my	is, by, at	go, or, so	up
5	on	be, of, no	us, so, do	as



11	5	2, 7, 4	3, 8, 9	6
12	5	7, 3, 6	8, 4, 9	2
13	8	5, 2, 4	6, 3, 9	7
14	7	5, 8, 3	4, 2, 9	6
15	9	3, 7, 4	5, 6, 2	8

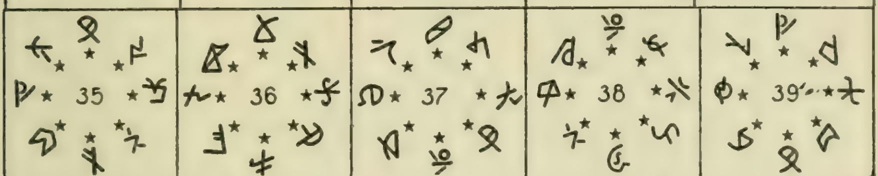


21	as	my, if, an	or, of, he	go
22	if	my, it, to	us, we, do	an
23	is	do, no, if	to, we, go	at
24	no	am, up, be	so, at, we	it

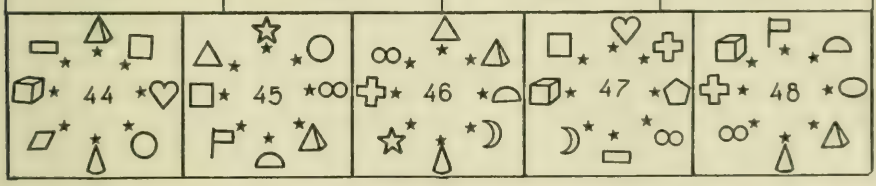
FORM A

Memory Tests (Page 2)

30	96	48, 53, 58	20, 67, 15	63
31	91	44, 49, 54	74, 68, 16	11
32	78	66, 97, 41	60, 65, 73	85
33	30	17, 61, 93	31, 99, 56	36
34	27	33, 40, 89	83, 28, 46	59



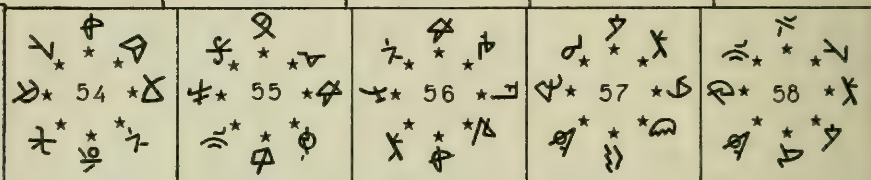
40	own	son, way, why	got, put, but	eat
41	get	cut, air, has	say, now, she	how
42	had	him, has, got	out, pay, did	she
43	his	pay, the, any	not, are, its	day



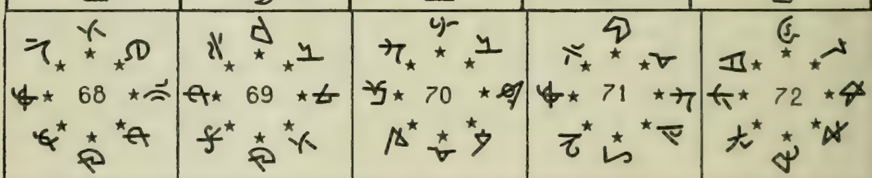
FORM A

Memory Tests (Page 3)

49	525	978	692	430	240	882	787	335
50	216	669	835	121	383	573	931	478
51	812	264	431	717	979	169	526	622
52	307	365	771	423	655	481	132	539
53	685	337	224	982	576	630	443	279



59	tell	soon	rest	live	does	fill	that	poor
60	hope	hear	said	late	such	from	when	wish
61	wait	dear	rich	were	each	glad	with	pass
62	done	lost	want	went	save	will	move	fine



FORM A

Memory Tests (Page 4)

73	74	75	76
carry	begin	early	heavy
begin	about	visit	about
given	never	being	given
other	along	happy	other
early	other	about	today
again	again	order	until
along	visit	alone	never
alone	heavy	often	cover

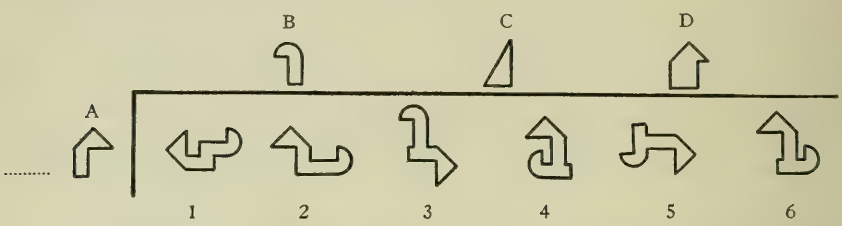
FORM A

Spatial Relationships Power Test (Page 1)

Name .... Date...
(First name) (Last name)

DIRECTIONS FOR FIGURE EXERCISE No. 2

We have here a figure exercise very much like the one that we gave you some time ago, but now we will give you much more time. We do not want you to work as fast as you can, but we want you to be sure to get every exercise right.



Look at Figure 1. It is made up of Figure A, and one of the figures B, C, and D shown at the top. Which figure at the top, together with Figure A, will make Figure 1? Yes, Figure B and Figure A together will make Figure 1. Write the capital letter B on the dotted line just to the left of Figure A.

Now I want you to draw a line in Figure 1 so that it cuts Figure 1 up into Figures A and B. Draw the line now Does either of these two parts need to be turned over in order to make Figure A or Figure B? Neither part needs to be turned over, so I want you to write the number 0 just above Figure 1. (Illustrate, with figure, on blackboard.) The 0 means that no part needs to be turned over.

Look at Figure 2 and draw a line in it so that it makes Figures A and B. Look carefully. Does the left-hand part need to be turned over to make Figure A? Yes, it does. Does the right-hand part need to be turned over to make Figure B? No, it does not, so in Figure 2 there is one part that must be turned over. You are therefore to make the number 1 just above Figure 2.

Look at Figure 3. When a line is drawn cutting it into the two figures, do either of these parts have to be turned over to make Figure A or Figure B? No, so you will write 0 just above Figure 3.

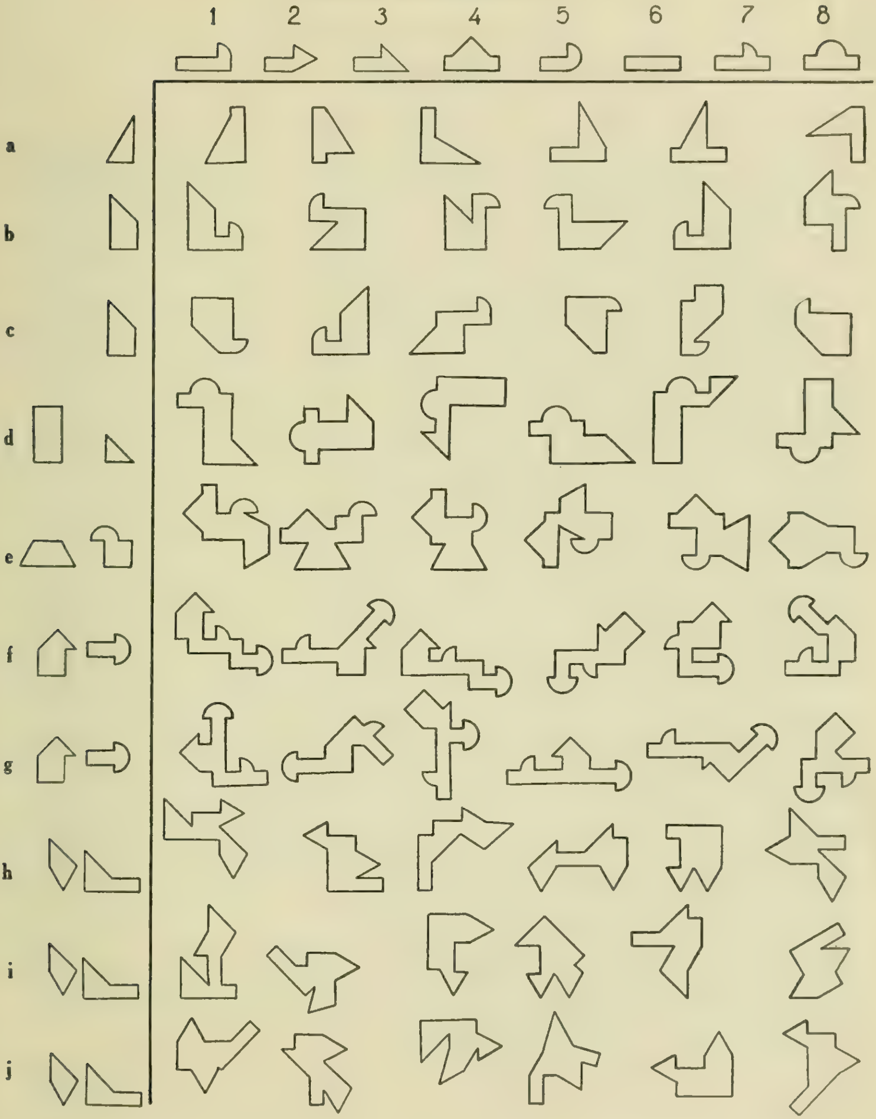
Now, look at Figure 4, and draw a line properly cutting it into two parts and write the number showing the number of parts that must be turned over. What number did you write? "2" is correct, for both parts must be turned over before they will exactly fit upon Figures A and B.

Do Figures 5 and 6 in the same way. (Give individual assistance where necessary.) You should have written 1 just above Figure 5 and 1 just above Figure 6, so that the numbers written in by you should now read: 0, 1, 0, 2, 1, 1. How many have them all right?

FORM A

Spatial Relationships Power Test (Page 2)

FIGURE EXERCISE No. 2



FORM B

Booklet of Speed Tests (Page 2)

(For directions see Form A, page 1)

READING EXERCISE

ON THE FARM

One day Walter and his younger (brother, sister) Helen went to visit their Grandpa's big (farm, city) They were very happy, for they knew that they would have a (cold, fine) time. When they got there, Grandma kissed them and gave them each a (cookie, whipping), saying, "Now go and play, children." On the farm there were many cows (but, and) horses. There was also a small brook which (stood, ran) tumbling along over the rocks down in the (road, meadow). At one place the brook grew (wider, taller) and became a little pool where many ducks and geese were (swimming, walking) about. Walter's Grandpa also had chickens and turkeys, but of course they could not (swim, walk) like the ducks. Back of the brook were some apple (vines, trees). Walter stood on his tiptoes to reach for a branch and pick an (acorn, apple). When Helen saw what he was doing she said, "(Walter, James), do not pick that because it is green and not good to (cook, eat)." They soon found two ripe (red white) apples, and they ate these because they were hungry in spite of having had a (cookie, sandwich). How good they tasted! Then Walter wanted to see what was inside of the big red (ship, barn) which was near the back of the house; so he and Helen went through the wide (world, doors) and climbed up a tall ladder to the top floor of the (house, barn). Everywhere they looked were great piles of hay for feeding the horses and (cattle, camels). In one corner they heard a strange (sight, noise). When they looked, they saw a big brown bird with staring (wings, eyes). Helen was afraid, but Walter shouted boldly, and the large (bird, cat) flew to the top of the barn. When the (children, boys) told Grandma about the bird they saw, she said it was a (brown, white) owl.

Then Grandma told (Tom, them) to go out to the little house built for the dog and (see, ask) what they would find. The children had often played with Flossie, the lovely shepherd (dog, lamb), when visiting the farm, and so were delighted to pay her a (dollar, visit) in her own little house. But when they reached it they found not only (Bessie, Flossie), but six wee black puppies which were scrambling (at, over) their mother and over each other. With their bright eyes and little noses, they (are, were) doing their best to learn all about the strange world in which (they, we) lived.

Flossie at first seemed a little doubtful about (asking, letting) the children handle her babies, but when she saw how (gentle, brutal) the children were, and felt, as dogs do, that there was (no, only) love in their feeling for the puppies, she let the children pick (apples, them) up and pet them.

Then Walter and (Constance, Helen) asked if they might go down to (sail, wade) in the brook, and Grandma gave them a towel with which to wipe their feet when they finished (swimming, wading). The water was very (shallow, rough), and not very cold, and the children saw some little fishes which would later grow (big, small) and would find their way to the deep river into which the (ocean, brook) emptied. They found, also, some funny long-tailed tadpoles which would later lose their tails and become (green, yellow) frogs, and would hop about on dry land as joyfully as they now (hopped, played) about in the water.

The children were so interested in the many kinds of (life, sea-shells) which they found in the water that they were greatly surprised when they (saw, heard) the deep tones of the big bell calling all the people on the (boat, farm) to come to the midday dinner. They then found that they were as hungry as Flossie's (chickens, puppies) and scampered back to the (barn, house).

Go right on to the next story and do the same thing.

FORM B

Booklet of Speed Tests (Page 3)

INDIANS

Have you ever seen a real, live Indian? Surely some of you (went, have) and all of you have seen pictures of them, and have (heard, seen) them in the movies.

Not so many years (ago, ahead) there was not a white person in this whole land of ours. There were only (black-skinned, red-skinned) Indians who wandered from place to place, living by hunting with a bow and (gun, arrow), and fishing with hooks made of bone. The skins of the animals which (they, we) killed were made into clothing and into tents to live (in, with). But most of the Indians have gone, and now we do not see many (in, of) them, except in the circus where they ride (trains, horses) at a great speed.

In the southwest part of the United (people, States) there are still a good many Indians left. Here it is very (hot, dark) and sandy, and it is not easy to get water. The sun shines nearly every (day, night) in the year. There are not many fish and wild animals, so the (Japanese, Indians) have learned to plant and grow a little corn. When this (corn, wheat) has grown they grind it and make it into large flat cakes, and then bake these cakes on (hot, cold) stones. The houses of the Indians are made of stone and mud with (empty, flat) roofs and small doors, and frequently one house is built right on top of another. In this case the Indians must climb (down, up) a ladder to get to their homes. Inside the rooms there are bright rugs which the Indians themselves have (made, found). There are also strong jars of clay, made hard by baking in the (river, fire) and baskets woven of grass so tightly that water can be carried (on, in) them. This country is so dry that there is very little (water, sunshine) and it must frequently be carried a long way and kept for a long time. That is why these Indians have learned to (buy, make) their jars and baskets so strong.

Years ago these Indians were known as bold fighters, but now they do not (fight, eat) any more. They are much happier staying at home in their strange houses and (listening, talking) to old tales.

As the tribes steadily decrease in (wages, numbers) and are forced to live far from their (former, future) homes, their customs and arts are passing (against, from) memory. The art of making the beautiful baskets and (china, blankets) may soon be forgotten. Some of the Indians are so poor that they cannot now (own, sell) sheep. Sometimes the women go far out on the (ocean, ranges) and gather small pieces of wool from the thorny shrubs which have torn it from the (cattle, sheep) as they passed. In former days the wool was colored with vegetable (dyes, paint) which did not injure it. Now they frequently use poor dyes, which fade and the blankets are (often, never) poorly made.

The Indians have many beautiful songs which were never written down but have (lived, died) only in their memories. Some of these have quite recently been written and printed by our own (carpenters, musicians). Some are hymns which were sung at daybreak to greet the (setting, rising) sun, and some are lullabies which put the (Japanese, Indian) babies to sleep. The Indian mother believes that when her child (goes, refuses) to sleep its soul passes into another world which is guarded by the spirits of the moon and the planets (and, in) the stars. So the mother prays to these spirits to guard and instruct her (child, mind) while it is in their care.

FORM B

Booklet of Speed Tests (Page 4)

ARITHMETIC EXERCISES

Add.

				3	8	4	5
2	7	8	9	7	5	3	4
6	4	7	6	9	2	7	9
8, 10	11, 13	20, 15	15, 17	19, 21	12, 15	14, 16	16, 18

Add·

6	7	8	2	5	7	9	8
2	9	5	9	8	4	9	9
6	3	9	7	8	8	6	7
14, 16	19, 21	22, 24	20; 18	23, 21	19, 18	26, 24	24, 25

Subtract:

12	15	17	18	13	19	14	16
6	5	8	7	4	5	7	9
6, 8	10, 5	9, 11	8, 11	9, 7	14, 12	9, 7	7, 5

Subtract:

16	12	14	11	15	18	19	19
8	7	5	8	3	9	6	8
6, 8	3, 5	9, 8	3, 4	11, 12	9, 11	11, 13	11, 9

Multiply:

5 × 2 = 10, 15	3 × 5 = 17, 15	4 × 2 = 6, 8	3 × 3 = 9, 12

Multiply.

3 × 2 = 8, 6	4 × 3 = 12, 14	5 × 4 = 20, 25	5 × 5 = 15, 25

Divide:

8 ÷ 2 = 4, 2	4 ÷ 2 = 1, 2	6 ÷ 3 = 2, 3	8 ÷ 4 = 4, 2

Divide:

10 ÷ 2 = 5, 2	6 ÷ 2 = 3, 2	9 ÷ 3 = 5, 3	10 ÷ 5 = 3, 2

Add:

2	4	7	8	2	5	2	3
7	2	6	5	9	8	9	5
5	8	4	6	8	6	5	7
3	2	3	7	4	6	7	8
17, 19	17, 16	20, 18	26, 28	25, 23	24, 25	23, 25	23, 27

Go right on with the problems on page 5.

FORM B

Booklet of Speed Tests (Page 5)

Add:

7	2	6	5	7	9	9	8
8	7	5	2	4	8	9	9
5	3	8	9	8	7	8	9
9	8	3	8	4	6	4	7
27, 29	20, 22	24, 22	28, 24	23, 21	30, 33	29, 30	35, 33

Subtract:

20	29	23	28	22	24	28	27
12	14	18	13	19	15	16	14
8, 6	15, 13	7, 5	15, 17	3, 4	9, 8	12, 14	15, 13

Subtract:

22	25	23	21	26	27	26	29
17	12	15	11	17	18	19	16
7, 5	13, 15	7, 8	9, 10	9, 7	11, 9	5, 7	17, 13

Multiply:

$2 \times 6 = 16,\ 12$ $5 \times 2 = 10,\ 8$ $3 \times 5 = 15,\ 10$ $4 \times 3 = 8,\ 12$

Multiply:

$3 \times 7 = 21,\ 18$ $3 \times 6 = 19,\ 18$ $8 \times 4 = 32,\ 36$ $9 \times 5 = 45,\ 54$

Divide:

$15 \div 3 = 5,\ 4$ $16 \div 4 = 4,\ 6$ $12 \div 3 = 3,\ 4$ $18 \div 9 = 2,\ 3$

Divide:

$20 \div 2 = 10,\ 5$ $14 \div 7 = 2,\ 6$ $16 \div 8 = 4,\ 2$ $18 \div 3 = 6,\ 7$

Multiply:

$8 \times 6 = 56,\ 48$ $6 \times 7 = 42,\ 49$ $8 \times 8 = 68,\ 64$ $6 \times 6 = 36,\ 39$

Multiply:

$9 \times 6 = 45,\ 54$ $7 \times 9 = 69,\ 63$ $7 \times 8 = 56,\ 57$ $8 \times 9 = 78,\ 72$

Divide.

$45 \div 15 = 3,\ 6$ $36 \div 18 = 5,\ 2$ $33 \div 11 = 3,\ 6$ $40 \div 10 = 5,\ 4$

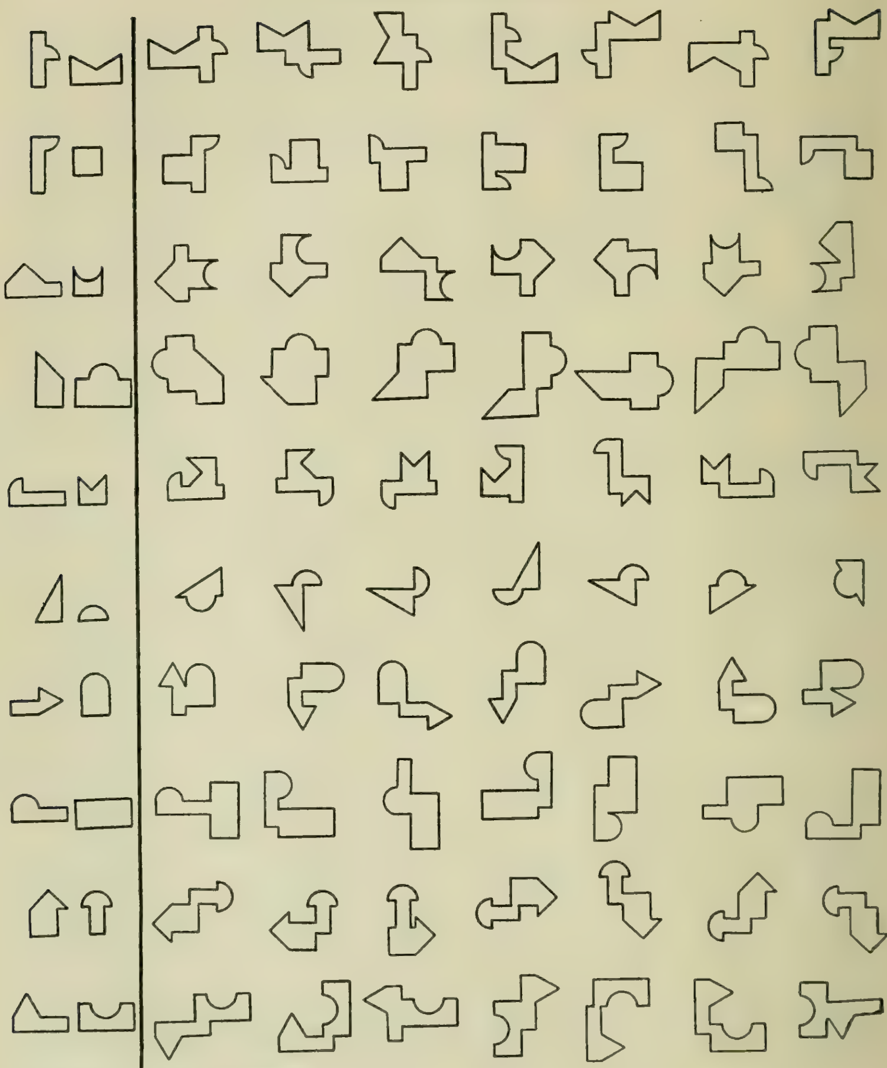
Divide:

$48 \div 16 = 4,\ 3$ $32 \div 16 = 3,\ 2$ $42 \div 14 = 2,\ 3$ $34 \div 17 = 4,\ 2$

FORM B

BOOKLET OF SPEED TESTS (PAGE 6)

FIGURE EXERCISE. No. 1

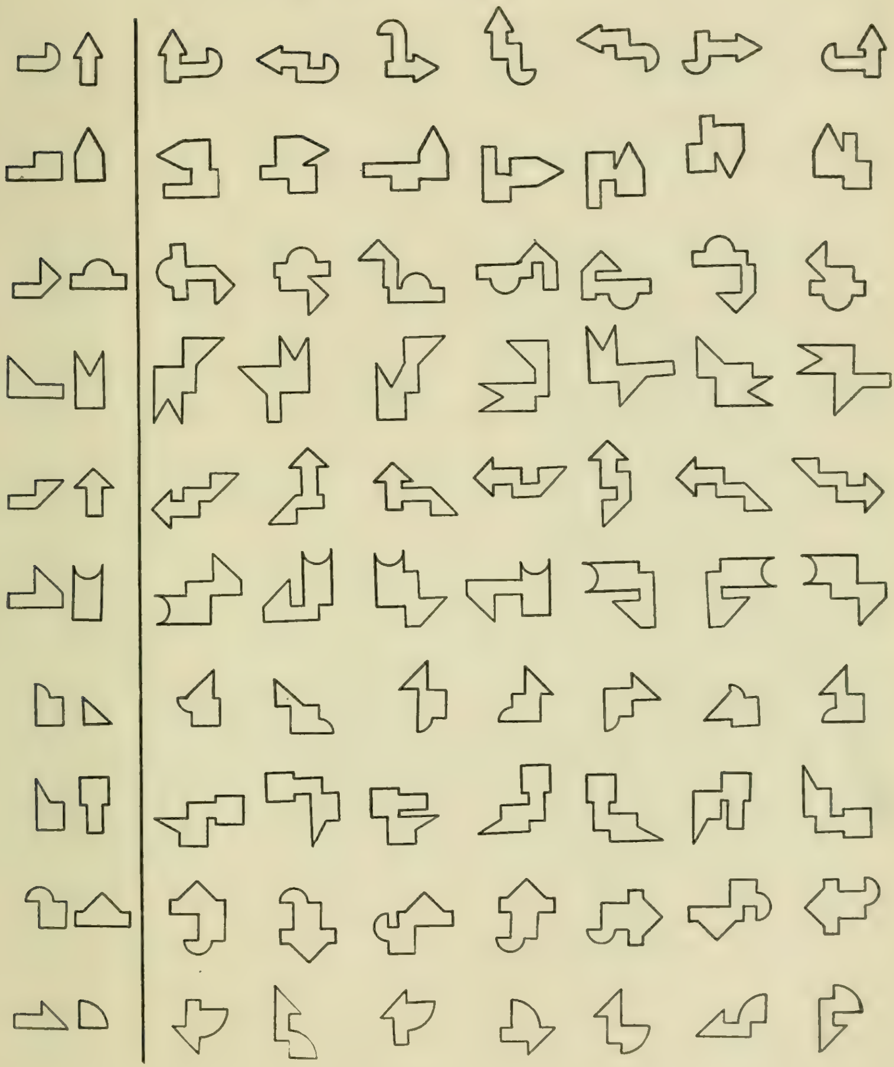


Go right on to the next page.

FORM B

Booklet of Speed Tests (Page 7)

(For directions see Form A, page 8)



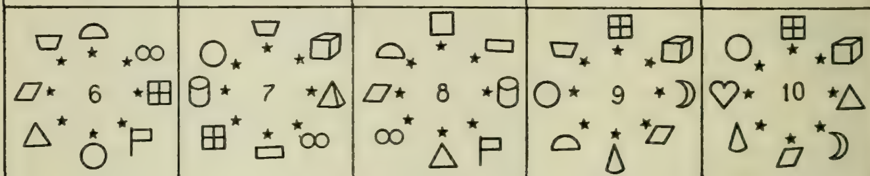
FORM B

Memory Tests (Page 1)

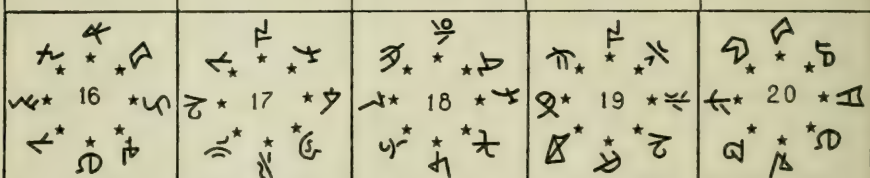
CIRCLES TEST

Name...
First name Last name

on ★ do ★ ★ be to ★ 1 ★ my go ★ ★ an ★ it	be ★ do ★ ★ on no ★ 2 ★ as he ★ ★ or ★ we	an ★ he ★ ★ so at ★ 3 ★ up am ★ ★ it ★ do	as ★ be ★ ★ at he ★ 4 ★ is to ★ ★ of ★ go	is ★ or ★ ★ us do ★ 5 ★ at am ★ ★ it ★ as



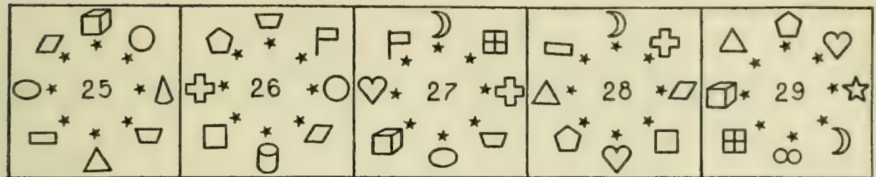
2 ★ 3 ★ ★ 9 7 ★ 11 ★ 5 8 ★ ★ 6 ★ 4	2 ★ 6 ★ ★ 7 8 ★ 12 ★ 4 9 ★ ★ 5 ★ 3	3 ★ 6 ★ ★ 2 4 ★ 13 ★ 7 5 ★ ★ 9 ★ 8	3 ★ 2 ★ ★ 7 6 ★ 14 ★ 5 9 ★ ★ 4 ★ 8	5 ★ 9 ★ ★ 7 2 ★ 15 ★ 3 6 ★ ★ 8 ★ 4



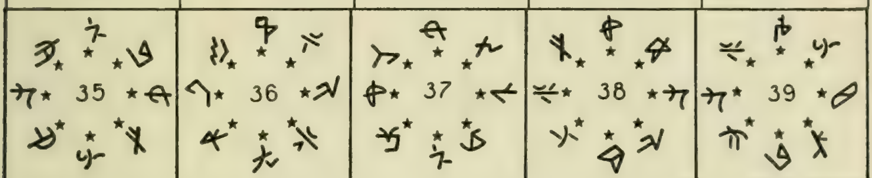
go ★ if ★ ★ of my ★ 21 ★ up we ★ ★ no ★ am	or ★ my ★ ★ on so ★ 22 ★ if no ★ ★ us ★ is	be ★ an ★ ★ to it ★ 23 ★ so in ★ ★ if ★ us	so ★ he ★ ★ as up ★ 24 ★ we or ★ ★ of ★ on

FORM B

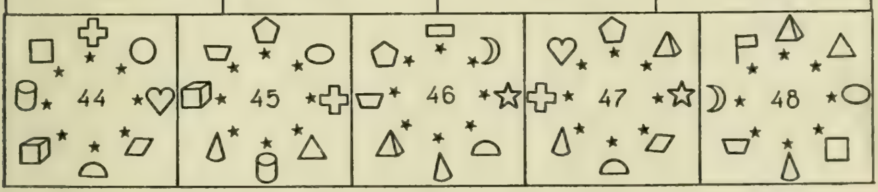
MEMORY TESTS (PAGE 2)



30	72	25, 77, 29	39, 87, 34	82
31	32	90, 64, 22	70, 12, 38	80
32	92	35, 79, 23	86, 42, 98	55
33	75	18, 50, 94	13, 51, 21	37
34	88	95, 45, 69	47, 71, 26	84



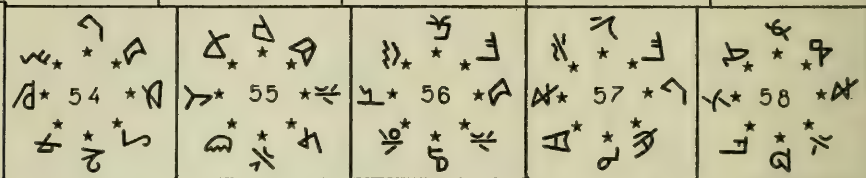
40	who	off, can, any	hot, our, was	ask
41	law	war, out, son	let, you, old	her
42	and	die, had, law	sit, too, her	yet
43	new	may, buy, for	too, use, hot	ask



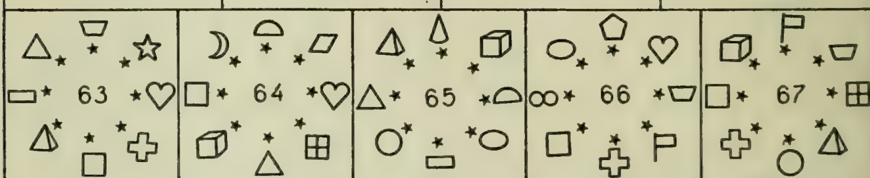
FORM B

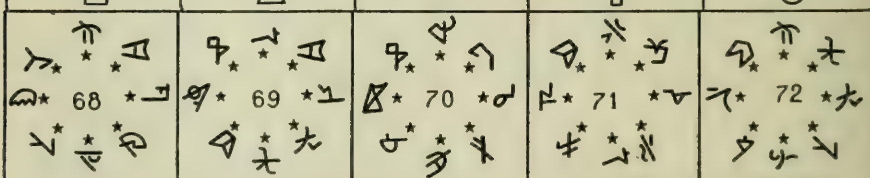
Memory Tests (Page 3)

145	288	884	829	873
597★ ★ ★764	740★ ★ ★360	336★ ★ ★249	945★ ★ ★388	521★ ★ ★412
502★ **49** ★311	193★ **50** ★907	788★ **51** ★140	497★ **52** ★729	170★ **53** ★735
954★ ★ ★859	645★ ★ ★455	241★ ★ ★115	146★ ★ ★537	818★ ★ ★927
407	550	958	794	467



read	draw	care	true
mean★ ★ ★word	show★ ★ ★name	life★ ★ ★keep	love★ ★ ★have
than★ **59** ★call	soft★ **60** ★came	hold★ **61** ★your	then★ **62** ★need
good★ ★ ★last	know★ ★ ★free	sure★ ★ ★made	seem★ ★ ★take
must	best	been	help

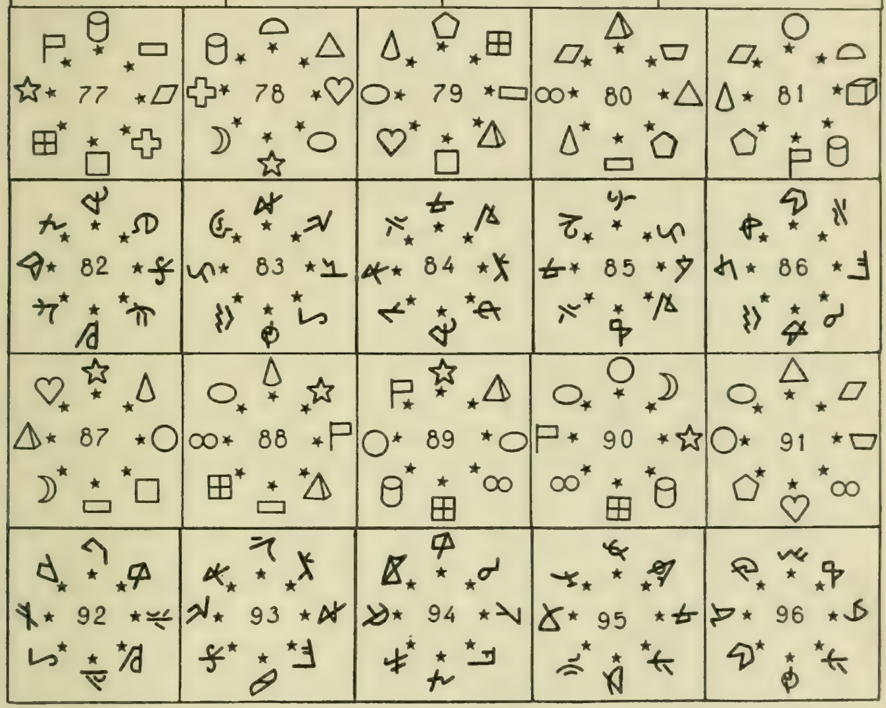




FORM B

Memory Tests (Page 4)

story money ★ ★ begin after ★ 73 ★ again carry ★ ★ until early	along alone ★ ★ until story ★ 74 ★ order often ★ ★ today money	given never ★ ★ happy today ★ 75 ★ after carry ★ ★ order being	often story ★ ★ heavy money ★ 76 ★ being after ★ ★ happy visit

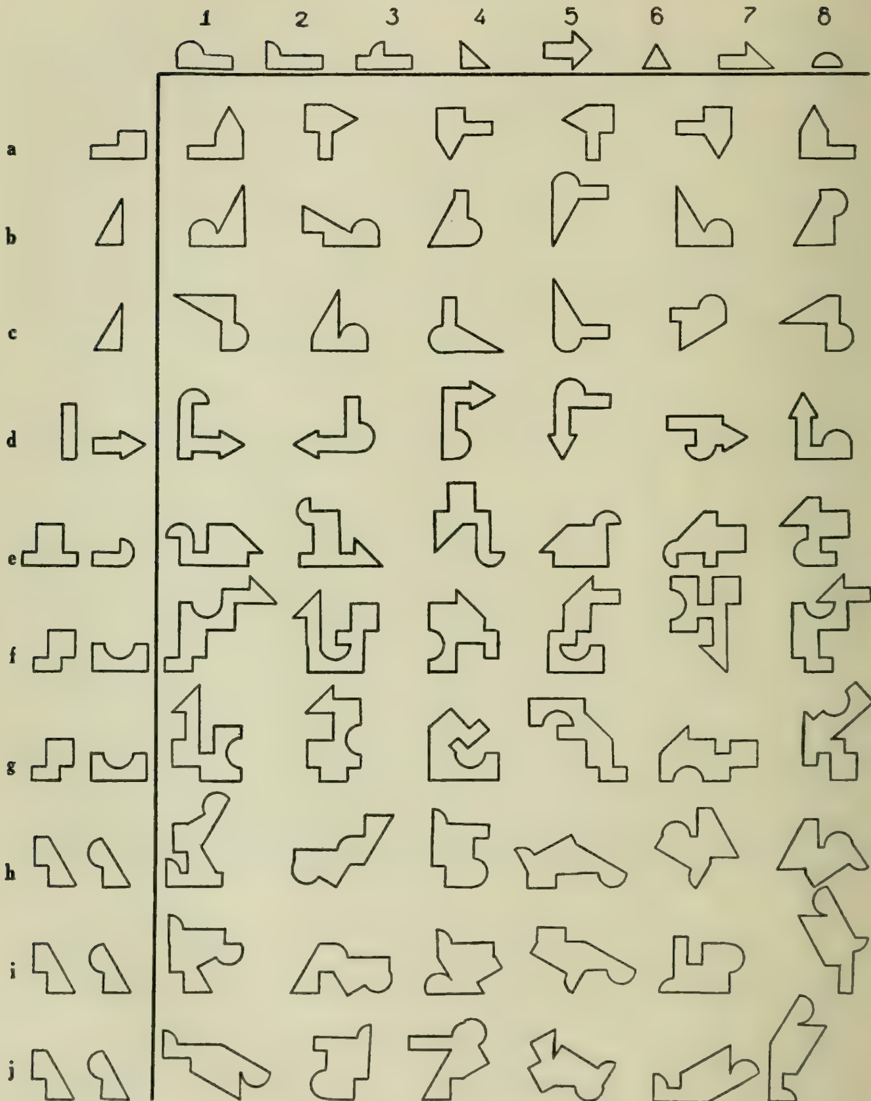


FORM B

Spatial Relationships Power Test (Page 2)

(For directions see Form A, page 1)

FIGURE EXERCISE No. 2



CHAPTER IX

MENTAL FACTORS REVEALED IN PUBLISHED STUDIES

Considering the universality with which psychological workers calculate correlation coefficients one would suppose that a large amount of data would be available in the literature for study by the methods here utilized. This, however, is not the case, because there are many requirements that need to be met before the data are valuable for the purpose of differentiating between abilities. It is highly desirable that the following conditions hold:

a) The population studied should be large. A reference to the formulas giving the probable errors of tetrad differences and of pentad functions shows the necessity of this. The writer considers that practically none of the data provided by Spearman and his students is amenable to the pentad function method or the more detailed methods of treatment herein presented, because of the small populations dealt with. It is of course true that these studies were made before the probable error formulas in question were available. Had they been available, Spearman would undoubtedly have worked with larger populations. If one assumes one single factor to be adequate in the explanation of a given set of inter-correlations, obviously the smaller the population the more likely are the data to be consistent with the assumption. It seems to the writer that Spearman's analyses have been largely affected by this kind of situation. The present writer's failure to analyze in detail the many published correlation tables of Spearman and his students is not due to unfamiliarity with them, for many of these studies have been carefully examined and found wanting for the present purpose because of the small populations involved, and because of failure to meet desideratum (*b*) given in the next paragraph.

b) The population should be as homogeneous as possible from the standpoint of general maturity, race, sex, and general scholastic training, otherwise any general factor found will be due in whole or in part to maturity, etc.

c) The measurements employed should be defined in terms of objective scores on designated tasks. Teachers' judgments or other subjective scoring schemes cannot be tolerated in this connection, because of the impossibility of objectively defining a function which is so measured.

d) The measurements employed should be of fairly discrete mental traits, for a measure involving, let us say, both reading and arithmetic will not enable us to differentiate between these two abilities. We thereby lose the possibility of studying the very issue that concerns us. Not infrequently fairly discrete tests have been given to subjects, but in reporting results omnibus scores only, generally sums or averages, have been recorded.

e) The separate measures employed should have high reliabilities. A test like the National Intelligence Test or like Army Alpha can be split up into several sub-tests, but the reliabilities of these sub-tests are very low. We accordingly cannot use the sub-tests on account of low reliability nor the total test score on account of the complexity of the function measured.

f) It is necessary that many variables be studied at a time.

In spite of the very general inappropriateness of the correlation data ordinarily published, it has seemed worth while to report on some of the most promising of the extant data.

In this connection we may use the very carefully reported study of Dr. Rose G. Anderson (1925). Her study has the following characteristics: (*a*) The population studied is large, being 382. (*b*) The population is homogeneous in the matter of age, as all the subjects were thirteen years old. It is heterogeneous in the matter of sex, and it is to be presumed in the matter of race. (*c*) All measurements are clearly defined in terms of specific tests. (*d*) A hasty examination of the various tests suggests that they are not at all discrete one from another. It was not Dr. Anderson's purpose that they should be. (*e*) The reliabilities of the separate tests must be rather low, as the reliability of the total battery is only .93 and the range of talent wide. (*f*) The number of variables studied is sixteen, which would be considered exceptionally adequate were it not for the fact that many of the tests measure nearly identical capacities. In spite of the shortcomings of these data they seem to the writer to be the most adequate of any that have been found in the psychological literature. The variables involved are as follows:

1. A letter-number-substitution test.
2. A test involving the selection of a pair of opposites of words.
3. A test involving the selection of three co-ordinates of words.
4. A test involving the designation of a word as a subordinate.
5. A test involving the selection of the one of five words not co-ordinate with the other four.
6. A verbal-directions test based on the alphabet.
7. A directions test involving the perception of separate letters in verbal material and reaction with a number.
8. A computation test involving the designation of the operations needed to obtain a given answer.
9. A verbal test involving the designation of predicates.
10. A directions test involving geometrical figures and the understanding of verbal statements concerning the same.
11. A number-series test.
12. A test involving elementary computation and following of directions.
13. A test involving the perception and retention over a short interval of the letters contained in words.
14. An arithmetic-reasoning test.
15. A disarranged-sentences test.
16. A test involving the supplying of a missing number in an arithmetic operation.

The reader is directed to the original monograph for more detailed descriptions of these tests and for the basic table of inter-correlations. Every test was correlated with every other and with the criterion. Using these inter-correlations by a method practically the same as that described on pages 199–200, it was ascertained that a number of the tests measured substantially the same function. Such tests have been combined into single variables by the present writer yielding eight variables whose inter-correlations are given in Table XXXVII.

TABLE XXXVII

INTER-CORRELATIONS FOR POPULATION OF 382 THIRTEEN-YEAR-OLDS

Variables	1	2	3	4	5	6	7	8
1								
2	.4527							
3	.4058	.5319						
4	.3193	.4108	.3961					
5	.3704	.4171	.5311	.4980				
6	.5267	.6499	.6255	.4410	.6406			
7	.5251	.5275	.5417	.4264	.6501	.6786		
8	.5124	.4800	.5436	.2968	.5367	.5722	.6265	

The variables of this table are related to those in Dr. Anderson's monograph as follows:

ABBREVIATED TITLE	VARIABLE TABLE XXXVII	DR. ANDERSON'S VARIABLE
Letter-number substitution	1	1
Directions—Verbal	2	6
Directions—Verbal and Number	3	7
Directions—Geometrical and Verbal	4	10
Arithmetic Reasoning	5	14
Verbal	6	Equally weighted average of Nos. 2, 3, 4, 9, and 15
Correction of imperfect and incomplete texts	7	Equally weighted average of Nos. 8, 11, and 13
Numbers test, involving elementary computation and the correction of incomplete operations in elementary computation	8	Equally weighted average of Nos. 12 and 16

From the correlations of these eight variables, given in Table XXXVII, tetrad differences were determined. The tetrad differences coming from Table XXXVII were credited to bonds which could conceivably have given rise to them as indicated in Table XXXVIII.

TABLE XXXVIII

Tetrad Differences in Population of 382 Thirteen-Year-Olds Allocated to the Pairs of Variables from Which They Conceivably Could Have Arisen

1, 2	1, 3	1, 4	1, 5	1, 6	1, 7	1, 8	2, 3	2, 4	2, 5	2, 6	2, 7	2, 8	3, 4
.01	.00	.01	.03	.02	.03	.03	.03	.00	.02	.01	.02	.02	.01
.01	.03	.02	.02	.02	.07	.08	.02	.02	.03	.02	.05	.03	.01
.07	.06	.02	.02	.01	.02	.08	.03	.02	—.07	.03	.03	.03	.03
.04	.00	.02	—.04	.03	.05	.06	.07	.01	—.03	.06	.09	.12	.01
.02	.03	.05	—.07	.01	.02	.03	.03	.02	—.09	.07	.05	.02	.04
.00	.06	.04	—.05	.02	.03	.07	.08	.05	—.02	.08	.02	.05	.04
.03	.02	.04	—.10	.05	.06	.08	.05	.08	—.07	.02	—.03	.07	.08
.05	—.01	.05	—.02	.01	.04	.02	.10	.06	—.02	.08	—.07	.05	.03
.09	—.07	.03	—.06	.05	.04	.08	.00	.05	—.08	.02	—.02	.03	.02
.07	—.03	.03	—.03	.06	.02	.03	.02	.02	—.03	.05	—.05	.02	.03
.02	—.02	—.04	—.06	.10	.08	.05	.08	.00	—.10	.05	—.10	.02	.02
.05	—.02	—.01	—.02	.01	.05	.07	.00	.01	—.05	.06	—.02	—.08	.03
.07	—.03	—.00	—.03	.09	.04	.09	.12	.07	—.08	.14	—.03	—.06	.05
.10	—.07	—.09	—.05	.01	.03	.03	.07	.03	—.08	.04	—.06	—.06	.00
.08	—.05	—.02	—.06	.07	.01	.02	.06	.08	—.12	.14	—.01	—.03	.03
.06	—.08	—.01	—.08	.03	.05	.08	.03	.09	—.05	.08	—.02	—.01	.04
.03	—.02	—.02	—.02	—.00	.10	.15	.01	.00	—.06	.11	—.01	—.08	.09
.03	—.03	—.05	—.07	—.07	.01	.04	.03	.10	—.03	.04	—.07	—.03	.02
.01	—.04	—.06	—.04	—.02	.01	.07	.00	.00	—.08	.11	—.03	—.02	—.05
.03	—.05	—.03	—.10	—.03	.09	.02	.05	.05	—.14	.08	—.02	—.03	—.02
.01	—.08	—.01	—.05	—.06	.05	.06	.07	.04	—.09	—.02	—.05	—.03	—.10
—.03	—.02	—.00	—.10	—.01	—.08	.12	—.01	.02	—.09	—.02	—.00	—.00	—.05
—.03	—.02	—.04	—.06	—.08	—.03	.05	—.00	.10	—.10	—.01	—.04	—.05	—.00
—.01	—.05	—.03	—.15	—.04	—.02	.10	—.04	.05	—.12	—.05	—.04	—.07	—.02
—.02	—.01	—.06	—.09	—.03	—.04	.05	—.00	—.01	—.14	—.00	—.10	—.03	—.01
—.02	—.05	—.05	—.09	—.01	—.06	.02	—.02	—.07	—.05	—.00	—.05	—.02	—.03
—.02	—.05	—.08	—.07	—.03	—.01	.05	—.02	—.05	—.11	—.01	—.08	—.05	—.06
—.08	—.09	—.01	—.12	—.07	—.05	—.03	—.07	—.00	—.07	—.00	—.03	—.04	—.01
—.03	—.03	—.01	—.05	—.05	—.03	—.01	—.03	—.06	—.02	—.04	—.10	—.08	—.06
—.07	—.02	—.02	—.10	—.02	—.05	—.01	—.05	—.02	—.05	—.02	—.02	—.05	—.01
Sum .57	—.74	—.30	1.71	.07	.53	1.58	.71	.76	—1.99	1.12	—.70	—.36	.16
M. . .02	—.02	—.01	.06	.00	.02	.05	.02	.03	—.07	.04	—.02	—.01	.01

TABLE XXXVIII—*Continued*

	3, 5	3, 6	3, 7	3, 8	4, 5	4, 6	4, 7	4, 8	5, 6	5, 7	5, 8	6, 7	6, 8	7, 8
	.05	.02	.03	.05	.09	.01	.02	—.08	.05	.10	.06	.03	.01	.03
	.04	.00	.01	.05	.07	.01	.00	—.02	.07	.08	.03	.02	.03	.01
	.02	.02	.02	.05	.03	.04	.01	—.05	.03	.06	.02	.05	.00	.04
	.02	.05	.02	.03	.06	.07	.02	—.08	.04	.05	.06	.01	.04	.03
	.05	.04	.01	.01	.06	.02	.06	—.06	.09	.09	.07	.03	.02	.05
	.02	.09	.01	.07	.10	.01	.00	—.15	.12	.01	.05	.00	.04	.07
	.08	.03	.04	.03	.05	.05	.02	—.03	.05	.10	.06	.05	.05	.10
	.05	.02	—.03	.05	.10	.01	.04	—.07	.08	.05	.03	.02	.10	.05
	.07	.00	—.00	.05	.08	.05	.05	—.04	.00	.12	.10	.00	.04	.11
	.05	.06	—.04	.03	.15	.03	.03	—.06	.08	.07	.11	.03	.01	.08
	.08	.08	—.06	.06	.05	.07	.07	—.07	.05	.09	.04	.01	—.07	.09
	.05	.05	—.08	.11	.10	—.01	.04	—.03	.07	.00	.02	.06	—.05	.02
	.03	.06	—.04	.01	.06	—.02	.04	—.10	.02	.14	.05	.05	—.09	.04
	.01	.03	—.01	.05	.14	—.02	—.02	—.12	.04	.08	.03	.06	—.04	.08
	.04	.08	—.00	.07	.09	—.03	—.04	—.04	.01	.05	.05	—.03	—.07	.02
	—.03	.02	—.12	.04	.02	—.04	—.05	—.02	.03	.03	.04	—.01	—.12	.08
	—.05	.03	—.05	.01	.12	—.10	—.10	—.10	.04	.03	—.05	—.05	—.03	.12
	—.00	—.01	—.01	.02	.06	—.02	—.01	—.05	—.02	.05	—.08	—.01	—.05	.03
	—.07	—.03	—.07	.06	.08	—.02	—.09	—.05	—.06	.06	—.05	—.01	—.05	.07
	—.03	—.05	—.03	.06	.01	—.08	—.09	—.11	—.01	.02	—.05	—.09	—.11	.03
	—.02	—.08	—.03	—.03	.04	—.14	—.00	—.04	—.06	.06	—.02	—.09	—.07	.04
	—.08	—.08	—.03	—.03	.06	—.04	—.05	—.05	—.08	.01	—.03	—.14	—.10	—.03
	—.03	—.03	—.09	—.02	.11	—.04	—.03	—.09	—.04	.01	—.06	—.05	—.02	—.02
	—.04	—.02	—.07	—.07	.05	—.02	—.04	—.07	—.06	—.02	—.01	—.08	—.01	—.02
	—.05	—.00	—.01	—.03	.06	—.02	—.00	—.10	—.01	—.05	—.04	—.05	—.03	—.05
	—.11	—.05	—.06	—.06	.05	—.08	—.02	—.05	—.06	—.01	—.06	—.02	—.04	—.10
	—.05	—.00	—.04	—.03	.10	—.03	—.06	—.12	—.06	—.05	—.05	—.08	—.08	—.02
	—.03	—.04	—.06	—.05	.08	—.03	—.01	—.04	—.01	—.12	—.08	—.04	—.06	—.02
	—.04	—.05	—.08	—.02	.12	—.05	—.08	—.08	—.10	—.04	—.04	—.04	—.03	—.05
	—.02	—.01	—.06	—.02	—.00	—.05	—.03	—.04	—.05	—.03	—.01	—.01	—.03	—.12
Sum.	.03	.23	—.93	.52	2.19	—.47	—.32	—2.01	.25	1.04	.19	—.42	—.81	.86
M.	.00	.01	—.03	.02	.07	—.02	—.01	—.07	.01	.03	.01	—.01	—.03	.03

In view of the none too satisfactory nature of the data involved, it hardly seemed warranted to spend the time that would be required to determine the most reasonable factor values for each of the variables. We thus concern ourselves with the first step only, (*a*), of chapter iii, page 80, and in fact will not do more than locate the variables in pairs between which lie special bonds. The procedure is that involved in chapter iii, pages 72–75.

In order to interpret the data of Table XXXVIII we need at least a rough idea of the size of the probable error of the mean tetrad differences. Having this we may then attempt to pick out pairs of variables between which lie a special bond. Two standard errors have been calculated as explained in chapter iii. The mean of the tetrad differences involving variables x_5 and x_7 is .03 and its standard error is .015. Assuming standard errors of the same general order for the other mean tetrad differences, we find some eleven situations in which the mean tetrad difference is more than two times its probable error. Let us seek for an explanation of these situations:

The mean tetrad difference involving variables x_4 and x_5 is .07. The nature of the special bond that is operative here is very obscure, but it apparently has something to do with the following of directions.

The mean $t_{4\,8\,i\,j} = -.07$. It is quite probable that three small negative bonds, verbal, spatial, and numerical, combine to give this fairly large negative value.

The mean $t_{2\,5\,i\,j} = -.07$. This is probably due to a negative numerical bond.

The mean $t_{1\,5\,i\,j} = -.06$. This is probably due to a negative verbal and numerical bond.

The mean $t_{1\,8\,i\,j} = .05$. This is probably due jointly to a numerical and a memory bond.

The mean $t_{2\,6\,i\,j} = .04$, and the mean $t_{2\,4\,i\,j} = .03$. These may both be due to verbal bonds.

The mean $t_{3\,7\,i\,j} = -.03$. The cause of this is quite uncertain.

The mean $t_{5\,7\,i\,j} = .03$. This is probably due to a numerical bond.

The mean $t_{6\,8\,i\,j} = -.03$. This may well be due to a negative verbal bond.

The mean $t_{7\,8\,i\,j} = .03$. It is likely that this is due to a numerical bond.

Though the results of our analysis of Dr. Anderson's data are generally corroborative of the findings earlier reported, there are still one or two bonds suggested for which we can offer no very probable explanation.

The next several reports upon other studies which are given herewith are modifications of class papers presented by the students mentioned.

Mr. J. W. Dunlap examined the data provided by Dr. Herbert A. Toops in "Tests of Vocational Fitness of Children" (1921), with the following results.

Cases studied: 44 boys of age fourteen.

Tests given:

1. Manual training
2. Stenquist Assembly
3. Stenquist Picture Completion, No. 1
4. Stenquist Picture Completion, No. 2
5. General Trade Test
6. Mechanical Interest Test

TABLE XXXIX

Table of Raw Correlations

Test	1	2	3	4	5	6
1. Manual Training Test						
2. Stenquist Assembly Test	.23					
3. Stenquist Picture Com., No. 1	.15	.60				
4. Stenquist Picture Com., No. 2	.05	.31	.71			
5. General Trade Test	.32	.26	—.02	—.04		
6. Mechanical Interest Test	.16	.52	.25	.37	.66	

The bonds suggested are as follows:

a) Stenquist Picture Completion Test, No. 1, x_3, and Picture Completion Test, No. 2, x_4. The only obvious difference between

TABLE XL

SUMMARY TABLE OF TETRAD DIFFERENCES

Variables in Pairs	No. of Positive Tetrads	Median Tetrad When Positive	Variables in Pairs	No. of Positive Tetrads	Median Tetrad When Positive
1, 2.....	6—1		2, 6....	6—1	
1, 3.....	6+0		3, 4....	6+6	.20
1, 4.....	6—2		3, 5....	6—3	
1, 5.....	6+4	.10	3, 6....	6—2	
1, 6.....	6—1		4, 5....	6—6	
2, 3.....	6+2	.07	4, 6....	6+2	.03
2, 4.....	6+0		5, 6....	6+4	.11
2, 5.....	6+0				

these two tests is that the second is harder than the first. We may therefore look upon this bond as due to specific elements in the Picture Completion Tests and credit the bond to common content.

b) Manual Training score, x_1, and General Trade Test, x_5. Here again the two tests are very similar in subject-matter, so that the bond may be looked upon as due to knowledge of a specific subject (tools, carpentry, etc.).

c) There are bonds between the General Trade Test, x_5, and the Mechanical Interest Test, x_6; also between the Stenquist Assembly Test, x_2, and the Stenquist Picture Completion Test, x_3; and finally between the Stenquist Picture Completion Test, No. 2, x_4, and the Mechanical Interest Test, x_6. As with reference to the other bonds, it seems reasonable to think of these as due to common subject-matter, i.e., they are bonds due to a specific content.

Mr. Herbert Popenoe has examined the data reported by Miss Francis Gaw in "A Study of Performance Tests" (1925).

Subjects tested: 52 boys just graduating from an English elementary school.

Tests employed: Miss Gaw gave seventeen tests, but because of the complexity of the work when dealing with such a large number of variables, and because of the small population tested, this number has here been reduced to eight. These eight tests were selected by dividing the average inter-correlation of each test by the estimated reliability coefficient of the test and then choosing for retention those tests yielding the smallest quotients. This process tends to select the tests with high reliability and much independence of each other. A brief description of the eight tests chosen for further study is given herewith.

1. Binet.

2. Dearborn Form Board. The test utilizes a form board containing six different types of figures or insets of simple geometrical shapes. In arranging the insets, there are numerous different combinations possible. The subject is given three problems, in each of which he must work out the minimum rearrangement of the blocks inside the board necessary to make room for one or more blocks outside the board. Scoring is based on time and on number of moves.

3. Healy Small Form Board A, the recesses of which are to be filled with blocks of various geometrical shapes. Score is time taken.

4. Goddard Adaptation Board. Oblong board through which four circular holes have been cut. Three are 6.8 cm. in diameter and the fourth 7.0 cm. A wooden block exactly fits the larger hole. Examiner illustrates that block will fit only one hole, and then turns board in five different positions. Scoring is number of times first placing of block by subject is in correct hole.

5. Healy Picture Completion Test, No. 1. Picture of 12 people doing various things in an outdoor scene. Ten holes have been cut in the picture, removing objects essential to the actions. Subject is given a collection of insets, including the missing objects and some others. Score is number of places filled correctly.

6. Porteus Maze Test. This test consists of seven mazes, graded in order of difficulty and standardized. Score is based on number of mazes passed and the number of trials necessary for success at each year.

7. Healy Picture Completion Test, No. 2. This test is similar to No. 1, except that it is more difficult. Scoring is as in No. 1.

8. Cube Imitation Test. Four black cubes are placed in a row before the subject, and with a fifth cube the examiner taps the four in a given order. There are twelve such patterns of increasing

difficulty, which the subject is required to imitate in turn. Scoring is based on number of patterns correctly imitated.

The correlations for these eight tests are given in Table XLI, and the summary table giving data upon tetrad differences is presented in Table XLII.

TABLE XLI

INTER-CORRELATIONS BETWEEN TESTS IN POPULATION OF 52 PUPILS COMPLETING THE ELEMENTARY SCHOOL

Test	1	2	3	4	5	6	7
1. Binet IQ...............							
2. Dearborn Form Board..	—.01						
3. Healy Form Board A...	.08	.05					
4. Goddard Adaptation Board	.32	.10	.29				
5. Healy Picture Completion Test No. 1.........	.18	.18	—.05	.34			
6. Porteus Maze..........	.52	—.21	.02	—.18	.13		
7. Healy Picture Completion Test No. 2.........	.57	—.28	.15	.32	.34	.11	
8. Cube Imitation Test....	.27	.13	.27	.21	.03	.12	.22

TABLE XLII

SUMMARY OF TETRAD DIFFERENCES IN POPULATION OF 52 PUPILS COMPLETING THE ELEMENTARY SCHOOL

Variables in Pairs	No. of Positive Tetrads	Value of Median Tetrad if Positive	Variables in Pairs	No. of Positive Tetrads	Value of Median Tetrad if Positive
1, 2.....	15+1	.01	3, 5....	15—7	
1, 3.....	15—2		3, 6....	15—1	
1, 4.....	15+4	.02	3, 7....	15+3	.01
1, 5.....	15—8		3, 8....	15+5	.03
1, 6.....	15+10	.07	4, 5....	15+2	.00
1, 7.....	15+2	.02	4, 6....	15—8	
1, 8.....	15—7		4, 7....	15+7	.04
2, 3.....	15+6	.01	4, 8....	15+3	.01
2, 4.....	15—5		5, 6....	15+4	.01
2, 5.....	15+7	.04	5, 7....	15+3	.01
2, 6.....	15—8		5, 8....	15—1	
2, 7.....	15—8		6, 7....	15+1	.01
2, 8.....	15+6	.03	6, 8....	15+2	.01
3, 4.....	15—3		7, 8....	15—8	

The standard error of the mean tetrad $t_{2\,5\,i\,j}$ is .031. With this value as a cue to the probable errors involved, let us examine the bonds that are suggested. There seems to be a bond between the Binet IQ, x_1, and the Porteus Maze Test, x_6. In view of the heterogeneous material entering into the Binet Test, it seems quite impossible to suggest the nature of this bond.

There may possibly be a bond between the Dearborn Form Board score, x_2, and the Healy Picture Completion Test, No. 1, x_5. A similar motor manipulation is involved in each of these.

There may also be a spatial factor leading to the slight x_2x_8 bond. A slight negative spatial factor may account for the x_3x_5 bond. The score on the Dearborn Form Board is designated x_2, that on the Healy Form Board A, x_3, and that on the Cube Imitation Test, x_8. On the whole the inter-correlations are very low, and accordingly the probable errors are large. We are not justified in drawing any very definite conclusions in regard to factors whether general, special, or specific. This study clearly illustrates the need for large populations and reliable scores, for even after reducing the number of tests from seventeen to eight we still have very inconclusive results.

Miss Gaw provided data similar to that just reported for 48 girls of average age 13.5. The raw correlations are given in Table XLIII.

TABLE XLIII

INTER-CORRELATIONS BETWEEN TESTS IN POPULATION OF 48 GIRLS COMPLETING THE ELEMENTARY SCHOOL

Test	1	2	3	4	5	6	7	8
1. Binet IQ								
2. Dearborn Form Board	—.21							
3. Healy Form Board A	—.10	.05						
4. Goddard Adaptation Test	.35	.10	—.12					
5. Healy Picture Completion, No. 1	.43	—.48	—.02	.07				
6. Porteus Maze	.29	.01	—.04	.12	.07			
7. Healy Picture Completion, No. 2	.38	.03	.07	.02	.37	.08		
8. Cube Imitation Test	.39	—.24	—.03	.20	.56	.42	.31	

These data were examined in detail with negative results. There was neither confirmation of bonds suggested by correlations found in the case of the boys, nor were new bonds suggested of sufficient certainty to call for mention. As can be surmised by reference to the correlations, the probable errors present are so large as to obscure any underlying tendencies.

Mr. Edward E. Cureton examined the data provided by Agnes L. Rogers in "Experimental Tests of Mathematical Ability and Their Prognostic Value" (1917).

Subjects tested: 61 girls entering high school, ages from 12 years 10 months to 16 years 11 months, and having a mean age of 14.6.

Tests employed: Seventeen tests were employed here, but the inter-correlations of eight only were selected, in the manner described on pages 199–200, for further examination.

Test	Function Tested
1. Matching *N*th Term and Series	Abstract symbol recognition
2. Geometry	Formal geometric logic
3. Superposition	Spatial relations in 2 dimensions
4. Symmetry	Spatial relations in 2 dimensions
5. Matching Solids and Surfaces	Spatial relations in 3 dimensions
6. Reasoning	Deduction
7. Mixed Relations	Word association and vocabulary
8. Logical Opposites	Word association and vocabulary

The inter-correlations, including reliability coefficients, are given in Table XLIV.

TABLE XLIV

Inter-correlations of Eight Tests in Population of 61 Girls Entering High School

(Coefficients along the diagonal are reliability coefficients)

Tests	1	2	3	4	5	6	7	8
1. Matching *N*th Term, etc.	**.89**							
2. Geometry	.24	**.86**						
3. Superposition	.11	.46	**.90**					
4. Symmetry	.14	.33	.61	**.90**				
5. Matching Solids, etc.	—.02	.45	.35	.33	**.82**			
6. Reasoning	.19	.43	.37	.28	.48	**.85**		
7. Mixed Relations	.08	.38	.23	.27	.31	.27	**.88**	
8. Logical Opposites	.28	.30	.14	.36	.21	.26	.26	**.73**

From this a summary table is obtained:

TABLE XLV

SUMMARY OF TETRAD DIFFERENCES IN POPULATION OF 61 GIRLS ENTERING HIGH SCHOOL

Variables in Pairs	No. of Positive Tetrads	Value of Median Tetrad if Positive	Variables in Pairs	No. of Positive Tetrads	Value of Median Tetrad if Positive
1, 2.....	15+ 9	.02	3, 5.....	15+ 3	.01
1, 3.....	15− 6		3, 6.....	15+ 5	.02
1, 4.....	15− 3		3, 7.....	15− 4	
1, 5.....	15−15		3, 8.....	15−13	
1, 6.....	15+ 4	.01	4, 5.....	15+ 2	.00
1, 7.....	15− 4		4, 6.....	15− 7	
1, 8.....	15+15	.07	4, 7.....	15− 1	
2, 3.....	15+ 3	.02	4, 8.....	15+ 2	.03
2, 4.....	15− 8		5, 6.....	15+ 6	.04
2, 5.....	15+ 0	.01	5, 7.....	15+ 8	.02
2, 6.....	15− 3		5, 8.....	15− 3	
2, 7.....	15+ 4	.02	6, 7.....	15− 6	
2, 8.....	15− 5		6, 8.....	15− 1	
3, 4.....	15+13	.10	7, 8.....	15+ 5	.03

The size of the mean tetrad involving variables x_1, matching the last term of an arithmetical series with a formula which yields for successive values of n the successive terms of the series, and x_8, the naming of logical opposites, is .07, and its standard error is .034. It does not seem reasonable to attribute this difference to chance. If Test 1, dealing as it does with the recognition of abstract symbols, is in large part verbal, then x_1x_8 bond is readily understood, as are also the negative bonds between the spatial tests and the verbal tests: x_1x_3, x_1x_4, x_1x_5, x_3x_8, and x_5x_8. Further, if Test 1 does possess a verbal element, that is, a bond with tests which we have earlier called verbal, we must immediately broaden or deepen our concept of the verbal factor.

To summarize the results of our study of Dr. Roger's data we can say: first, there is clearly indicated a spatial factor; second, though the tests were used to forecast ability in mathematics, the particular eight tests of Dr. Roger's seventeen which we have studied do not permit the proof or disproof of the existence of a numerical factor or of a memory factor; third, there is a suggestion that the verbal factor extends to tasks involved in the comprehension of symbols in a mathematical formula; fourth, a factor not earlier revealed, involving thinking in terms of antitheses, whether verbal or spatial, is suggested. This last is suggested by the x_4x_8 bond in view of the negative x_3x_8 bond and the positive x_3x_4 bond.

Mr. Cureton has also examined the data given by J. E. W. Wallin in "Consistency Shown by Intelligence Ratings Based on Standard Tests and Teachers' Ratings" (1922).

Subjects tested: 34 pupils in the first grade of the Miami University Practice School.

Measures employed:

1. Stanford-Binet Test
2. Pressey Primer Test
3. Myers Mental Measure
4. Detroit First-Grade Intelligence Test
5. Teachers' estimates of intelligence

TABLE XLVI

RAW CORRELATION COEFFICIENTS

Tests	1	2	3	4
1. Stanford-Binet Test				
2. Pressey Primer Test	.455			
3. Myers Mental Measure	.678	.379		
4. Detroit First-Grade Intelligence Test	.439	.508	.464	
5. Teachers' estimates of intelligence	.487	.380	.557	.297

We here have an x_2x_4 bond, which is not surprising in view of the similarity of these two tests.

Test 2: The Pressey Primer Scale is a picture test consisting of four parts: dot pattern, classifications of familiar objects, form board relations, and absurdities. The first is a test of symmetry recognition, the second of recognition of common objects, the third of matching geometrical forms, and the last of recognizing the absurdities in pictures of common objects. The responses are all the same, "Cross out the thing that does not belong."

Test 4: The Detroit First-Grade Intelligence Test consists of ten tests: recognition of materials [general information, as "Mark the things made of wood" in four pictures], recognition of identical geometric forms, recognition of common objects, recognition of absurdities, recognition of uses of objects ["Mark the things you use with a knife"], recognition of size relations, filling in spaces in symmetrical figures, picture completion, number ability ["Mark two of the chairs," in a picture in which three appear], and following directions.

This test is quite similar in its elements and types of response called for to Test 2, many of the tests being little more than duplicate forms of the same thing.

The omnibus nature of all these five measures makes it impossible to attribute bonds to detailed mental functions.

TABLE XLVII

SUMMARY TABLE OF TETRAD DIFFERENCES IN POPULATION OF 34 PUPILS IN FIRST GRADE

Variables in Pairs	No. of Positive Tetrads	Value of Median Tetrad if Positive	Variables in Pairs	No. of Positive Tetrads	Value of Median Tetrad if Positive
1, 2.....	3—1		2, 4.....	3+3	.12
1, 3.....	3+1	.04	2, 5.....	3+1	.02
1, 4.....	3+0		3, 4.....	3+0	
1, 5.....	3+0		3, 5.....	3+2	.06
2, 3.....	3—3		4, 5.....	3—3	

Mr. James E. McCormack has examined part of the data provided by Mr. H. G. Stead in "Factors in Mental and Scholastic Ability" (1926).

Subjects: 127 boys actually in an elementary school, and 8 boys who had just (at the time of testing) been admitted from an elementary to a secondary school. All of the boys tested had reached their twelfth year, but none had yet reached the age of thirteen.

Tests employed: Mr. Stead used four different types of measures: motor tests, character tests, scholastic tests, and psychological tests, only the last of which is here examined. The character measures are not sufficiently objective for our purposes, and the motor and scholastic tests yield such low inter-correlations as to suggest low reliability; in fact the psychological tests are none too promising.

Description of tests: Stead does not give an adequate description of the tests but refers the reader to Burt (1922) and to Ballard (1922). From an examination of the tests in both Ballard and Burt it is evident that Stead made his own forms, using the general types of tests described in the two books. His brief descriptions of the tests show that he did not take tests directly out of the books.

1. Completion Test (Usual continuous passage form of the Completion Test.)
2. Opposites Test
3. Instructions Test

Sample:

1. Put a dot under this line: ————
2. Cross out both A's in the word "ADA"
3. Make a girl's name by adding one letter to "Mar "

Burt says: "The various so-called 'Instructions' tests are based on the view that the measurement of a number of different mental activities provide a better test of intelligence than the measurement of only one activity. The questions here used have been roughly graded in order of increasing difficulty. Most of the questions indicate a type that might well be made the basis of a homogeneous series of questions, were it so desired."

4. Number Series Test
5. Synonyms Test
6. Cipher Test

It is very difficult to determine just what type of a Cipher Test Stead did use. Burt does not give any Cipher Tests and the test described in Ballard does not follow Stead's simple description.

Stead described the Cipher Test briefly by saying that it was composed of "five single words followed by four sentences." The test in Ballard's book involves a key which gives punctuation marks used to represent the different vowels and the consonate *h*. Using this code, Ballard then asks a number of questions which are to be answered by a single word. Ballard gave twenty-five such questions, scaled according to difficulty.

7. Absurdities Test

From Stead's brief description it would seem that he followed the type of test suggested by Ballard rather than the one used by Burt.

Instructions:

Here are 20 foolish statements. Under each statement you will find four reasons why the statement is foolish. Check the one which you think is best.

No time limit was imposed on this test.

Sample:

1. A soldier writing home to his mother said, "I am writing this letter with a sword in one hand and a pistol in the other." Foolish because
   - A. The pistol might go off.
   - B. He could not write with a sword.
   - C. He could not write with both hands occupied.
   - D. Perhaps his mother could not read.

8. Analogies Test

The basic correlations involving these eight variables are given in Table XLVIII and a summary of the tetrad differences therefrom is found in Table XLIX.

There is a bond between the Cipher Test, x_6, and the Number Series Test, x_4, for the mean t_{46ij} tetrad = .07, and its standard error is .028. Because of the uncertainty of the writer as to the nature of the Cipher Test, speculation as to the nature of this bond is rather profitless. About all one can say is that both tests involve a search for missing elements. In one case numbers are required to complete the meaning, and in the other vowels.

TABLE XLVIII

INTER-CORRELATIONS BETWEEN TESTS IN POPULATION OF 127 BOYS IN AN ELEMENTARY SCHOOL

Tests	1	2	3	4	5	6	7
1. Completion Test..........							
2. Opposites Test..........	.58						
3. Instructions Test........	.58	.47					
4. Number Series Test.....	.51	.50	.51				
5. Synonyms Test..........	.48	.53	.48	.34			
6. Cipher Test..............	.43	.40	.41	.50	.33		
7. Absurdities Test	.43	.44	.45	.35	.41	.41	
8. Analogies Test..........	.46	.45	.38	.40	.39	.33	.31

TABLE XLIX

SUMMARY TABLE OF TETRAD DIFFERENCES

Variables in Pairs	No. of Positive Tetrads	Value of Median Tetrad if Positive	Variables in Pairs	No. of Positive Tetrads	Value of Median Tetrad if Positive
1, 2.....	15+ 3	.01	3, 5.....	15+ 2	.01
1, 3.....	15+ 4	.02	3, 6.....	15— 2	
1, 4.....	15— 2		3, 7.....	15+ 6	.02
1, 5.....	15— 2		3, 8.....	15— 6	
1, 6.....	15— 1		4, 5.....	15—14	
1, 7.....	15— 5		4, 6.....	15+14	.07
1, 8.....	15+ 3	.01	4, 7.....	15— 9	
2, 3.....	15—10		4, 8.....	15+ 4	.01
2, 4.....	15+ 1	.00	5, 6.....	15—11	
2, 5.....	15+10	.03	5, 7.....	15+ 8	.02
2, 6.....	15—10		5, 8.....	15+ 7	.01
2, 7.....	15+ 1	.00	6, 7.....	15+11	.04
2, 8.....	15+ 5	.01	6, 8.....	15— 1	
3, 4.....	15+ 6	.01	7, 8.....	15—12	

There is apparently a bond between the Cipher Test, x_6, and the Absurdities Test, x_7. The nature of this is very uncertain. There is indicated a bond between variables 2, opposites, and 5, synonyms, which we may think of as verbal and perhaps also involving the antithesis bond suggested by Dr. Roger's data.

A very recent contribution emanating from Spearman's laboratory is the study of Hargreaves upon "The 'Faculty' of the Imagination" (1927). In this study the older criteria of hierarchical order and correlations between columns have been entirely discarded and a much sounder procedure based upon tetrad differences has been followed throughout. Though the present writer still feels the necessity of making a reservation as to the statistical soundness of one of Spearman's techniques which has been used, that of determining the standard deviation (or probable error) of the distribution of all the tetrad differences coming from a table of inter-correlations (see above, page 12), he nevertheless does not take exception to the detailed findings. In other words, if the formula giving the standard deviation is in error, it is probably not grossly in error, for conclusions reached by its use are much in line with those following a very different procedure (for example, that of previous chapters involving the standard error of a mean tetrad). Though Hargreaves' purpose is to determine the existence and nature of imagination, he provides data on a large number of other points.

One point upon which we particularly would have liked information is almost entirely neglected, for he scarcely considers the effect of maturity, racial, and sex differences on measures of correlation. All he writes in regard to his group is as follows: "The tests were given to five classes (two of boys and three of girls) in three schools, A, B, C. The total number of children was 200, but absences reduced these ultimately to 151. Their average age was 12.8." From this we have no means of determining how large a factor maturity is. The question does not seem to have occurred to Hargreaves.

Dr. Hargreaves finds a small "fluency" factor, which is measured by the following tests: A—Unfinished Picture Test, B—Unfinished Stories Test, C—Writing Words (the number of disconnected words written in three minutes).

He also finds a speed factor which he surmises to be more than merely speed of writing, though it is found in the two following tests: A—Speed in Writing Figures, B—Speed in Copying Prose.

After quoting Lankes, Wynn Jones, and Bernstein as having found evidence of a "preservation" factor, he states that there is similar evidence to be gathered from certain of his tests: (*a*) Reversed Stroke Test. (Figures 2 3 4 5 6 7 were written rapidly and repeated for thirty seconds. They were then written in the reverse order both as to each figure and as to the order of the figures. The score is a measure of the relative efficiency of the reverse writing in comparison with the direct.) (*b*) Inverted *S* Test. (Subjects wrote both *S* and a reverse *S* according to a designated arrangement. The score is similar to that for the Reversed Stroke Test.) (*c*) *I T* Test. (A given paragraph was copied rapidly for two minutes. It was then copied again with the instructions that the *i*'s were not to be dotted nor the *t*'s crossed. The score is a function of the accuracy and the speed with which this was done. The reliability proved to be very low, .41.)

Dr. Hargreaves' finds evidence to attribute speed and another unknown factor to "absence of inhibitions." He concludes that such speed is not operative in power tests.

Having found these various factors he considers that they quite adequately account for fluency of imagination. Therefore it is not necessary to postulate such a factor as over and above those already mentioned. He writes the equation:

$$\text{Fluency of Imagination} = g + \text{Speed} + \text{Memory} + x$$

in which g is the general factor, and Speed and x are conative, depending upon the absence of inhibitions.

He also states: "When imagination tests are marked for originality, denoting by this the extent to which a subject's ideas are novel or uncommon, the imagination (originality) tests have some common group factor, but very little relationship to g." This factor is compound, and if in part separate from other factors mentioned, the magnitude of this part is small.

This monograph refers to certain results of other workers which have bearing upon our general problem. Burt is quoted as having found that "linguistic ability and attainments exert upon the Binet-Simon Tests a special and positive influence of their own." Schwegler (1920) and Winn are quoted as having found that, though colored children do not exceed white children in any test, they do equal them in (1) common-sense adjustment to practical situations, (2) rote memory tests not requiring manipulation of material, (3) tests involving primary verbal facility. Since the populations tested were very small and almost undoubtedly far from constituting random age groups, we can attach but little significance to these results.

The data of Bonser (1910) are particularly interesting for our study because they have been repeatedly cited by Spearman, the last citation appearing in his *Abilities of Man* (1927) as giving evidence of the operation of a single general factor. Bonser's group in fact constitutes the only large population that Spearman does cite as supporting his view.[1]

In view of Spearman's criticism of other writers who provide data that do not support the general factor theory, because of their failure to properly take into account differences in maturity and sex (1927, p. 159), is it not peculiar that he cites Bonser's data (1910) at all at this late date, for this group of 757 is composed of 385 boys and 372 girls of an age range from eight to sixteen, found in the upper divisions of fourth, fifth, and sixth grades in five different public schools. To use the words of Spearman in criticizing others, surely "heterogeneity in age, sex, etc. . . . . has been allowed to run riot." If these things have not been uncontrolled in this population, what does constitute lack of control? Though Spearman has cited heterogeneity as a disturbing influence in the works of others, he seems really to have given very little thought to it. It seems but fair to charge him with lack of concern with maturity and sex as *causing* a general factor. The failure of his pupil Hargreaves is the most recent piece of evidence. Unfortunately this point of view has been operative in Spearman's early work as well. For reasons already given above,

[1] Though Spearman writes (1927, p. 238), ". . . . the tests employed by the present writer with some 30,000 candidates for the British Civil Service . . . . ," it nevertheless is obvious that his correlations and tetrad differences were not based upon this entire population, for he gives (*ibid.*, p. 239) a tetrad-difference probable error of .007, thus conclusively indicating that the population for this particular study was much less than 30,000.

in chapter ii, differences in maturity, sex, and race may be looked upon as quite sufficient to account for any general factor that may be found whenever these things are not taken account of. We may accordingly anticipate a large general factor in the Bonser data. Even so, the writer does not believe that this general factor is so large as entirely to obscure a second factor in these data. The following table is pertinent in this connection.

TABLE L

Inter-correlations between Tests in Population of 757, Consisting of 385 Boys and 372 Girls from the Upper Division of Grades 4, 5, and 6, from Five Public Schools, Ages 8–16

Tests	1	2	3	4
1. Mathematical judgment				
2. Controlled association	.485			
3. Literary interpretation	.400	.397		
4. Selective judgment	.397	.397	.335	
5. Spelling	.295	.247	.275	.195

It may be readily shown that when spelling is omitted the other four variables yield tetrads very closely equal to zero, and could therefore be readily explained as due to one general factor (g according to Spearman, and probably maturity and sex according to the present writer). Spelling, however, involves in addition to a large maturity factor special "memory" and "contentment with drill" factors (see Kelley, 1926) which form special bonds with certain of the other four variables. Let us consider the mean tetrads, together with their probable errors, involving x_5, the variable spelling:

$$\text{Mean } t_{1\,5\,i\,j} = .009 \qquad \text{P.E.}_{\cdot t} = .007$$

$$\text{Mean } t_{2\,5\,i\,j} = -.015 \qquad \text{P.E.}_{\cdot t} = .007$$

$$\text{Mean } t_{3\,5\,i\,j} = .025 \qquad \text{P.E.}_{\cdot t} = .008$$

$$\text{Mean } t_{4\,5\,i\,j} = -.019 \qquad \text{P.E.}_{\cdot t} = .008$$

Surely one should conclude that these four mean tetrads are not mere chance deviations from zero.

These data have been considered in detail, not because they are appropriate for the study of independent mental factors, but because they are based upon the only large population quoted by Spearman as supporting his view.

The writer feels that Spearman is guilty of one other serious oversight in addition to his lack of concern with maturity, etc., and that is a failure to appreciate that his point of view cannot be statistically established by a study of small populations. Doubly serious is this shortcoming if the instruments of measurement are not precise.

Let us illustrate this by an entirely different problem. A certain person tosses ten coins, upon the twenty faces of which are a heads and b tails ($a = 11$ and $b = 9$, but this fact is not known to the experimenter). These are tossed ten times with the result that there turn up a total of 55 heads and 45 tails. The experimenter maintains that the coins have ten heads, and he argues that since the standard error of the number of heads in this number of tosses is equal to 5, and since he got a number of heads, namely 55, which could readily have occurred as a matter of chance, there is no reason for him to revise his judgment, so he continues to maintain that the number of heads on the coins is 10. It must be obvious that if he does not increase his population he will never prove to his own satisfaction that the number of heads is not ten.

It would be particularly hard for one adopting Spearman's standard to disprove such a conviction once held, because Spearman demands an unduly extreme divergence from chance results before an accepted hypothesis is to be discredited. On page 295 of *Abilities of Man,* he writes:

> The tetrad difference does indeed have the high-looking value of .20, but even this becomes insignificant upon comparing it with the probable error, which is .13.[1] [Spearman's footnote] [1] The reader may be reminded that an experimental value should be at least three times larger than its probable error before it can be taken even as suggestive, and it must be five times larger before its evidence can be deemed conclusive. See pages 140–141.

Again on page 141 he writes in connection with the establishment of a conclusion:

> Most summary of all is to see whether or not the largest observed tetrad difference exceeds five times the magnitude of their probable error,

and again on page 238:

> The tetrad difference comes to .090 which is only three times its probable error, .030, and therefore is not conclusive.

This standard for determining significance seems to the writer very unsound because it is altogether too exacting. In brief, it seems to the writer that wherever there was one chance in two, or half a chance, or even a much smaller fraction of a chance, Spearman has interpreted results in favor of his theory.

The populations studied in detail in earlier chapters of this work are admittedly far too small and too heterogeneous to yield very adequate or true pictures. However, any such defect has operated to make the discovery of second, third, and fourth independent factors difficult, while at the same time it would tend to make appear quite adequate a single factor. In spite of heterogeneity and small population thus working against a multiple-factor hypothesis, the populations which were studied do not point to the sufficiency of a single general factor, so that it may well be believed that results from the study of still larger populations would be still less in harmony with the single-factor hypothesis.

The special study of the inter-correlations between the eight sub-tests of the Army Alpha Test, conducted by the Division of Psychology of the Surgeon General's Office (Yerkes, 1921), provides data upon a large population of American white English-speaking adults in nine different army camps. The special merits of these data are that they cover a large population, sex is constant, and maturity substantially constant. The defects are that racial homogeneity is not attained, the tests separately have low reliability, and a technique of treatment was followed which makes it impossible to calculate the probable errors of the correlation coefficients or of the tetrad differences. The number of cases examined was 1,047, and it was found that the distribution of a number of sub-tests gave undistributed extreme scores, zero, or perfect scores. Accordingly, all correlations were corrected for these undistributed scores, to obtain estimates of the correlation maintaining if the tests had permitted lower or higher scoring as required. Though the writer sees no reason to believe that these very carefully made corrections either introduced or took out any general or group factors, still it is true that the corrections are of such magnitude that ordinary probable-error formulas may not be used for the determination of reliability.

The sub-tests of the Army Alpha are:

1. Directions: Oral directions requiring the making of marks in geometric forms, etc., on the test blank.
2. Arithmetic-reasoning problems.
3. Practical Judgment: One of three choices of action to be indicated in each of 16 series.
4. Synonym-Antonym.
5. Disarranged Sentences.
6. Number Series.
7. Analogies.
8. Information: One of four alternative responses to be marked.

Tables LI and LII give the inter-correlations and a summary of the tetrad differences.

TABLE LI

Correlations between Eight Sub-Tests of the Army Alpha in Population of 1,047 Men Picked as a Special Experimental Group from the United States Army

Tests	1	2	3	4	5	6	7	8
1. Directions								
2. Arithmetic	.730							
3. Practical Judgment	.590	.745						
4. Synonym-Antonym	.710	.791	.805					
5. Disarranged Sentences	.686	.763	.754	.834				
6. Number Series	.680	.773	.613	.681	.674			
7. Analogies	.670	.736	.671	.730	.778	.704		
8. Information	.658	.742	.775	.861	.823	.693	.672	

TABLE LII

SUMMARY TABLE OF TETRAD DIFFERENCES IN POPULATION OF 1,047 WHITE MALE ADULTS

Variables in Pairs	No. of Positive Tetrads	Value of Mean Tetrad	Variables in Pairs	No. of Positive Tetrads	Value of Mean Tetrad
1, 2.....	15+ 5	.034	3, 6.....	15— 8	—.041
1, 3.....	15—14	—.062	3, 7.....	15— 3	—.015
1, 4.....	15+ 0	—.010	3, 8.....	15+ 7	.048
1, 5.....	15+ 0	—.016			
1, 6.....	15+ 5	.052	4, 5.....	15+ 0	.013
1, 7.....	15+ 9	.028	4, 6.....	15—11	—.054
1, 8.....	15— 5	—.028	4, 7.....	15— 4	—.030
			4, 8.....	15+11	.056
2, 3.....	15+ 7	.008			
2, 4.....	15— 5	—.026	5, 6.....	15— 7	—.042
2, 5.....	15— 7	—.036	5, 7.....	15+ 8	.040
2, 6.....	15+12	.058	5, 8.....	15+ 4	.028
2, 7.....	15— 5	—.002			
2, 8.....	15— 7	—.041	6, 7.....	15+ 7	.033
			6, 8.....	15+ 2	—.010
3, 4.....	15+ 9	.049			
3, 5.....	15+ 2	.013	7, 8.....	15—12	—.053

The standard errors of four mean tetrads have been calculated by Formula 64 with the result noted below, but the reader is warned not to place much trust in these values because of the curtailment of the original distribution and the consequent adjustments that were made.

$$t_{1\,3\,i\,j} = -.062 \qquad \sigma_t = .0075$$

$$t_{2\,6\,i\,j} = \ \ .058 \qquad \sigma_t = .0074$$

$$t_{4\,5\,i\,j} = \ \ .013 \qquad \sigma_t = .0062$$

$$t_{4\,6\,i\,j} = -.054 \qquad \sigma_t = .0053$$

After making considerable allowance for the uncertainty of our results it still seems that a number of important factors are indicated. The probable explanation of the bonds found are given in Table LIII.

TABLE LIII

BONDS SUGGESTED BY THE ARMY ALPHA DATA

Variables in Pairs	Mean Tetrad Difference	Probable Nature of Bond	Variables in Pairs	Mean Tetrad Difference	Probable Nature of Bond
1, 2....	.034	Numerical factor	3, 8....	.048	Verbal factor
1, 6....	.052	Numerical factor	4, 8....	.056	Verbal factor
2, 6....	.058	Numerical factor	5, 7....	.040	?
3, 4....	.049	Verbal factor	6, 7....	.033	?

The general nature of the factors suggested by this table is in considerable agreement with that deduced from previous studies.

CHAPTER X

FURTHER APPROACHES TO AND FUTURE OUTCOMES OF A DETERMINATION OF UNITARY MENTAL TRAITS

Though the utilization of judgments in the determination of mental structure is fraught with difficulty, it can be made to provide suggestions for objective measurement, and in its own right it provides a very fascinating means of studying the independence of mental concepts of judges. Dr. Shen (1924, *Experimental;* 1925, *Validity;* 1925, *Influence;* 1925, *Reliability*) has employed very effectively the judgments of acquaintances. The subjects consisted of 28 Chinese male students in American colleges, and judgments were made upon the following traits:

1. Intellectual Quickness = ability to think, understand, and learn rapidly.
2. Intellectual Profoundness = ability to think, understand, and learn thoroughly.
3. Memory = facility to remember or retain ideas correctly.
4. Impulsiveness = tendency to act without a thorough plan and take a chance.
5. Adaptability = ability to adjust oneself to new requirements, to new problems and conditions of life.
6. Persistence = ability to maintain a definite purpose until it is attained.
7. Leadership = ability to lead in any (except purely physical) organized activity, to inspire confidence and secure co-operation and support of one's colleagues.

All subjects were also judges, and as they had been classmates for from three to nine years, the intimacy of acquaintance was greater than is ordinarily the case in judgment studies. The technique was drawn up with a view to avoiding systematic errors. The basic table of inter-correlations is given in Table LIV:

TABLE LIV

CORRELATIONS OF FINAL COMPOSITE RATINGS

r_{12} Raw coefficients are in Roman type.
$r_{\infty\omega}$ *Corrected coefficients are italicized.*
$(\sigma_{r_{\infty\omega}})$ Standard errors are in parentheses.
All decimal points are omitted.

Traits	Intellectual Quickness	Intellectual Profoundness	Memory	Impulsiveness	Adaptability	Persistence	Leadership
Intellectual Profoundness	88						
	92						
	(04)						
Memory	95	93					
	99	*98*					
	(02)	(03)					
Impulsiveness	—21	—40	—28				
	—24	*—46*	*—32*				
	(21)	(18)	(21)				
Adaptability	54	31	37	—01			
	56	*33*	*39*	*—02*			
	(14)	(18)	(17)	(22)			
Persistence	49	77	60	—28	03		
	53	*78*	*64*	*—33*	*03*		
	(15)	(09)	(13)	(21)	(20)		
Leadership	66	79	65	—24	50	77	
	69	*83*	*69*	*—29*	*53*	*83*	
	(11)	(07)	(11)	(21)	(15)	(08)	
Scholarship	91	91	96	—24	37	62	64
	95	*95*	*100*	*—27*	*38*	*66*	*67*
	(03)	(03)	(02)	(21)	(17)	(12)	(11)

Dr. Shen has allowed for the chance element in the measures by using coefficients corrected for attenuation, and he has provided the standard errors of these corrected coefficients. Though the population is small, the unusual intimacy of acquaintance makes this study most interesting, whether looked upon as a study of the relationship of ideas in the minds of judges, or of the relationship of traits in people. Dr. Shen has kindly permitted the writer to quote from his dissertation at some length, and the writer is glad to do so because the dissertation is not in print. The reader can establish for himself the reasonableness of Dr. Shen's observations by an analysis of the raw data given in full in Table LIV.

The correlation [corrected for attenuation] between them [Intellectual Quickness and Intellectual Profoundness] though very high is more than two times the standard error below unity, justifying the postulation of a difference. . . . . According to our own results, the intellectually quick is more impulsive, more adaptable, but less persistent, than the intellectually profound. In fact, with these three traits constant, the partial correlation between quickness and profoundness reaches perfect unity. As our population is rather small, a partial correlation with three constants has a low reliability. None the less, it seems entirely reasonable that the difference between quickness and profoundness is due to such modifying factors. . . . . the difference between quickness and profoundness seems best explainable in terms of attention, interest, and attitude. Quickness means shifting attention, unspecialized interest, and alert attitude; the mind is set upon reacting upon the environment for immediate ends. Profoundness, on the other hand, means sustained attention, specialized interest, and careful attitude; the mind is set upon ignoring immediate distractions and reacting for more remote ends. On the negative side, the quick is often superficial and accomplishes little of importance, while the profound is sometimes hopelessly dull and stupid. The difference seems to be largely acquired and to a certain extent controllable. Everybody is quicker at one time and more profound at another. Mental tests are unable to find the difference because the subjects are instructed to work as rapidly as possible and to turn their attention from one item to another, leaving no freedom for their natural or habitual inclination. This at least partly explains why McQueen (1917) finds no general attention factor and Bernstein (1924) finds no general speed factor. While we feel entirely justified to recognize the difference, we do not mean to forget their more important similarity.

In regard to memory:

The perfect correlation of unity with scholarship indicates that the latter must have been almost the exclusive criterion, supplemented probably by a small amount of casual observations of incidental memory.

Impulsiveness is perhaps the most interesting of our eight traits. It has a negative correlation with all the other traits. Thomson (1924) and Thurstone (1921, 1924) have pointed out that inhibition is an important factor of intelligence, and our impulsiveness seems to be nothing other than the lack of inhibition. The highest negative correlation that impulsiveness has is —.46 with intellectual profoundness. If we now hold the latter constant and calculate partial correlations of impulsiveness with the other traits, the results become all positive, as follows:

With intellectual quickness	.53
With memory	.74
With adaptability	.16
With persistence	.05
With leadership	.18
With scholarship	.60

Apparently, then, with a given amount of intellectual profoundness, impulsiveness actually increases quickness, facilitates memory, makes an individual at once more adaptable and more persistent, as well as a better leader and a better scholar. The paradox is far from unreasonable. When impulses are regulated by intellect, it means energy, vigor, and activity without rashness. Intelligence held constant, more success naturally follows more trial and error. While intelligent behavior requires a certain amount of inhibition, inhibition cannot be substituted for intelligence.

The low correlation of .03 between adaptability and persistence apparently indicates that the two traits are independent. Closer analysis, however, will reveal that they are really supplementary in one sense and antagonistic in another. . . . . Withholding intelligence, adaptability becomes aimless change and persistence remains mere obstinancy. The partial correlation between them, with either intellectual quickness or intellectual profoundness constant, is —.38, showing that they are actually opposing tendencies which only intelligence can put into cooperation. It is probably due to this antagonism that character and intelligence are often contrasted. The adaptable may take as means what is commonly regarded as end and thus become cleverly unprincipled. The persistent, on the other hand, may seriously take a means as end and thus remain faithfully foolish. The contrast is, however, not justifiable so long as intelligence is able to overshadow the difference. With a given degree or amount of intelligence, the relative predominance of adaptability or persistence seems partly due to heredity and partly due to environment. McDougall (1921) has already pointed out that the black man is more adaptable while the red man is more persistent. A similar difference in milder degree is here tentatively suggested between the southerner and the northerner in China.

Leadership depends more upon intellectual profoundness than upon intellectual quickness, and more upon persistence than upon adaptability. Its multiple correlation with adaptability and persistence together is almost perfect, .97. This multiple correlation remains as high as .91, even when intellectual profoundness is held constant. Leadership, then, does not seem to require very high abstract thinking beyond a co-operation of adaptability and persistence.

While leadership and scholarship have a total correlation of .67, their partial correlation with intellectual profoundness constant is —.68. This is but natural, since both traits correlate high with intelligence, which is responsible for their positive correlation. With a given degree of intelligence, a better leader is a poorer scholar, and vice versa, though one may be intelligent enough to be good in both.

. . . . Our analysis shows that among them the intellectual qualities, impulsiveness, adaptability, and persistence are the most important, interesting, and fundamental traits. It seems that intelligent behavior is largely the regulation of impulses and the co-operation between adaptability and persistence, while intellect may involve a more abstract analytical capacity besides.

It has already been pointed out that elements of genetic structure, which are independent and at the root of intellectual life, could conceivably be selected by breeding, resulting in races showing substantial differences in the trait bred in or out. There is much casual observation which is very suggestive from this point of view and there is also considerable quantitative measurement. As it is beyond the scope of the present treatment to review these studies in detail, we will examine one only as a sample.

Dr. Darsie (1926) studies the mental capacities of American-born Japanese children. He in part controlled the language problem by examining such Japanese as were thoroughly familiar with English.

The investigation was limited to American-born Japanese children between the ages of ten and fifteen inclusive. Such children had in most cases never been out of America, they had attended English-speaking schools for at least four or five years, and with the exception of certain rural groups English was the language most familiar to them. Every effort was made to secure a group which would be thoroughly representative of the race in California.

The Stanford-Binet, the Stanford Achievement, and the Army-Beta Tests were given. A ten-point difference in Stanford-Binet IQ in favor of the American white child was found, but in spite of the care in selection it is reasonable to think that some though hardly all of this may be due to a language handicap arising from the fact that many of the Japanese were bilingual. This gross difference does not concern us nearly so much as does evidence of difference along detailed lines. Let us consider the separate tests involved.

The Beta results indicated equality of American and Japanese performance for ages below twelve. At the age of twelve the Japanese were appreciably superior. Table LV lists the various tests and offers tentative explanation of any marked differences found.

TABLE LV

Japanese-White Differences upon Army Beta Tests

Tests	Differences	Suggestions as to Certain of the Mental Factors Involved
Digit Symbol Test	Marked Japanese superiority	Numerical and memory
Number Comparison Test	Marked Japanese superiority	Numerical and memory
Picture Completion Test	Appreciable white superiority	Spatial relationships (1) and (2)
Cube Analysis Test	Slight Japanese superiority	
Maze	Slight white superiority	
Series Completion Test	Insignificant difference	

The results of Stanford Achievement Tests are summarized in Table LVI. Coarser age groupings are here employed than in Darsie's monograph.

TABLE LVI

JAPANESE-WHITE DIFFERENCES UPON STANFORD ACHIEVEMENT TESTS

Tests	Differences	Suggestions as to Mental Factors Involved
Reading	Marked white superiority	Verbal
Language Usage	Appreciable white superiority	Verbal
Spelling	Slight Japanese superiority	Verbal and memory
Arithmetic	Negligible difference	Verbal and numerical
Information Tests	Slight white superiority	Verbal, some memory and other factors

In Table LVII the separate tests of the Stanford-Binet are ranked in order of white superiority to Japanese. The population yielding these data was of the mental age twelve. The fact that the subjects are not of the same chronological age somewhat complicates the interpretation of the results. The figure in the second column gives the Japanese-White difference divided by its standard error.

TABLE LVII

JAPANESE-WHITE DIFFERENCES UPON ELEMENTS IN THE STANFORD-BINET TEST

Test Element	Difference / Standard Error	Suggestions as to Certain of the Factors Involved
1–XII–1. Vocabulary 40	(−10.50)	Verbal
2–X–1. Vocabulary 30	(− 7.92)	Verbal
3–XII–2. Abstract Words Defined	(− 4.73)	Verbal
4–XIV–1. Vocabulary 50	(− 4.44)	Verbal
5–XIV–Al 1. Seven Digits	(− 4.02)	Verbal, numerical, and memory
6–X–Al 1. Six Digits	(− 3.96)	Verbal, numerical, and memory
7–XVIII–4. Logical Memory	(− 3.53)	Verbal and memory
8–XVI–3. Differences between Abstract Words	(− 3.21)	Verbal

TABLE LVII—*Continued*

Test Element	Difference / Standard Error	Suggestions as to Certain of the Factors Involved
9–XII–4. Dissected Sentences	(— 3.13)	Verbal
10–XVIII–3. Eight Digits	(— 2.50)	Verbal, numerical, and memory
11–X–5. Comprehension Fourth Degree	(— 1.98)	?
12–XII–5. Fables 4	(— 1.79)	Verbal
13–XIV–6. Reversing hands of clock	(— 1.50)	Spatial relationships (1)
14–X–6. Sixty Words	(— 1.30)	Verbal
15–XVI–2. Fables 8	(— 1.11)	Verbal
16–XIV–4. Problems, Fact	(— 1.06)	Verbal
17–XIV–5. Arithmetic Problems	(— 0.31)	Verbal and numerical
18–X–2. Absurdities	(— 0.11)	Verbal
19.5–XII–6. Five Digits Reversed	(— 0.10)	Verbal, numerical, spatial, and memory
19.5–XII–3. Ball and Field	(— 0.10)	Spatial
*22–XVI–1. Vocabulary 65	(0.00)	Verbal
22–IX–6. Rhymes	(0.00)	Verbal
22–IX–3. Making Change	(0.00)	Verbal and numerical
24–XII–8. Similarities	(+ 0.41)	Verbal
25–X–4. Reading Memory	(+ 0.62)	Verbal and memory
26–IX–Al 1. Name of Month	(+ 0.99)	Memory
27–XII–7. Picture Description	(+ 1.16)	?
28–XIV–2. Induction	(+ 1.48)	Spatial relationships (2)
29–XIV–3. President and King	(+ 1.68)	Antithesis
30–XVI–4. Enclosed Boxes	(+ 1.92)	Spatial relationships (1) and (2)
*31–IX–5. Three-Word Sentences	(+ 2.09)	*
32–IX–2. Weights	(+ 2.49)	Kinaesthetic
*33–IX–1. Naming Days	(+ 2.50)	*
34–X–3. Copying Designs	(+ 2.57)	Kinaesthetic and manipulation of spatial relationships (1) and (2)
35–XVIII–2. Paper Cutting	(+ 2.66)	Manipulation of spatial relationships (2)
36–XVI–5. Six Digits Reversed	(+ 2.89)	Numerical and memory
*37–IX–4. Digits Reversed	(+ 2.89)	*
38–XVI–6. Code	(+ 5.70)	Memory and manipulation of spatial relationships (1) and (2)

* Unreliable because of insufficient data.

Dr. Darsie also reports that on the basis of the teacher's judgments Japanese are decidedly superior to American white children in penmanship, painting, and drawing, and inferior by about half as much in reading, composition, grammar, and literature. Also the Japanese are superior in "Appreciation of Beauty," "Permanence of Moods," "Freedom from Vanity," and "Conscientiousness," and they are inferior in "Amount of Physical Energy," "Self-Confidence," "Desire to Know," "Originality," and "General Intelligence."

A study of Table LVII points to a superiority of White to Japanese in verbal ability, a superiority of Japanese to White in kinaesthetic sense and probably also in memory and manipulation of spatial relationships, Spatial 2, of chapters vi and vii, but not in Spatial 1 of those chapters. The picture is not clear-cut and leaves much in the way of detail and certainty to be desired.

The recognition of the reality of differences between races in particularized mental traits cannot but affect individual and national outlooks as to racial admixture. If a certain race loves music showing graduations of pitch scarcely sensed by another or if it enjoys drawing and design in which occur contrast angles and intricate detail wasted on those who love sinuous lines and color harmonies, there is small wonder that the one race does not warm to the culture of the other. But the world is small and men must live together. If they can but live in understanding of one another, we may then expect them at least to show respect and tolerance toward each other. Western peoples have long thought that races differ one from another in terms of superiority. Though this is probably true with reference to any mental trait which is truly unitary, a more exact comparison of race with race would not be expected to reveal a dead level of difference in all mental traits. It is this detailed picture that is necessary before problems of migration and intermarriage can be intelligently approached. There is surely no natural law to the effect that all crosses are universally good, or universally bad.

Could the buzzard be crossed with the canary, we should not expect as an outcome either an efficient scavanger or a song bird—we might at best get something like a macaw, which is a virtueless bird in both respects. Were a thrush crossed with a canary, would the outcome be a poor mute bird or one delivering such harmonies as the ear has never heard? Unfortunately we do not know. If a cross between two human beings having outstanding talents results in offspring with nondescript virtues, it may be looked upon as disgenic. But if the outstanding talents can combine to make a still greater talent, surely the world has been served. The mutual recognition by the thinking fraction of two peoples of the truth in the matter of racial blending, coupled with the adoption of a policy in conformity with it, should result in an amity of feeling between them whether migration and intermarriage follow or not. These issues are not minor and the determination of the essential facts would be a fairly colossal undertaking, but the principles and methods to follow appear fairly within grasp and entirely straightforward. In brief, it would be necessary to determine what are the functionally independent mental traits; are they acquired or inherited, and, if the latter, how do they behave when crosses take place? Any light that a knowledge of independent mental traits would bring to the solution of racial eugenic problems would be equally and more immediately serviceable in connection with individual eugenics if the contracting parties were inclined to heed it.

Our present happy-go-lucky method of choosing life partners does result in musicians being more likely to marry musicians, of literary people being more likely to marry literary people, etc., than chance would give. Perhaps this tendency has had a cumulative effect, and through the operation of biological laws not as yet understood has been the cause of such unitary mental traits as now assuredly exist. But how inefficient the process: our great musicians do not tend to marry equally great musicians, nor our great literary men equally great literary women. The regression toward the mean in the choosing of mates is perhaps given by a correlation in the neighborhood of .2 or .3, and is thus so great as to be very ineffective. The writer does not wish to be understood to advocate mating of like with like at all times, but only so doing when it is established that the cross gives much promise of being an individual of high social value.

Another way in which a knowledge of independent mental traits can be of great individual value is in connection with the laws of learning. Dr. Blank was discouraged in his attempt to teach appreciation of literature because he said there was no foothold in the subject. To illustrate this he cited that he personally enjoys alliteration, and for a time thought he was doing good in the world when he pointed out alliterative niceties in prose and poetry. However, a not negligible number of his neophytes received his instruction with no sense of fulfilment of a felt need. Dr. Blank decided that if one liked that sort of thing he liked it, and if he didn't then he didn't. What has Thorndike's law of satisfyingness of response to do with this situation? Practically the law is useless until we know what constitutes satisfaction, and we will not know that until we know what trait is being satisfied and how. Surely the technique of accomplishing satisfaction, if not in fact the very basic principle itself, will change as we pass from verbal matter to spatial relationships. Not only this law but every law of learning so glibly memorized by students of pedagogy demands verification and citation in connection with the separate fields of mental functioning before it can hold a rich and thoroughly serviceable content for the teacher.

Considerable recognition has been given to independent mental traits in vocation, industry, and business. We now need a still more careful study of what these traits are and of the demands of each of the important vocations upon them. That there are widely different demands is axiomatic. A certain acquaintance of the writer was struck by the incompetence of salesmen when outside of the selling fields known to each. Such incompetence is probably no greater than that of other workers, each outside of his particular field, but we do not expect the accountant to be a public speaker, the machinist to be an accountant, or even the boss of the manufacturing plant to be an expert machinist. Though it is probably true that the verbal facility and comraderie of the salesman are very specialized traits, nevertheless they have a less concrete and measureable outcome than is the case with numerical, musical, or manipulative ability, and thus they are not so readily recognized. They are, however, fully credited in the practical administration of an insurance company, an automobile or bond sales force, etc. We may look to industry for practical applications of the independence of mental traits, and the psychologist will find therein one of the largest fields ready to utilize his sound analyses of mental structure.

In the academic world we are so remote from economic pressure and so little able to measure the actual outcomes of our instruction that we have a poor field in which to find everyday illustrations of the independence of mental traits. "Every candidate for the Doctor's degree should at least have a good command of English." How common is this remark, and how inadequate the analysis which leads to it. Is not its antithesis—"Every candidate should at most be able to do one thing exceptionally well"—intrinsically more reasonable? Student A, known to the writer, is brilliant in mathematical analysis but is nearly inarticulate in verbal expression. His contributions, written presumably with the help of some other person, are found in mathematical journals. This man cannot truly pass any minimal English requirements that would be proposed. Perhaps the requirements for the doctorate should be broadened: "Every candidate should be able at most to do one thing exceptionally well or to do a number of things with considerable merit." Certainly the first sort of individual should be recognized. The independence of mental traits is such that it is possible to go far along one line without commensurate progress along other lines, provided the "lines" are such as tax fairly unitary functions. If this specialized progress is great intellectually and valuable socially, should it not be recognized by our handclaps, Ph.D. degrees, and such like? Clearly we need to know far better than we now do what mental traits are thus self-conditioned; but the mere recognition of the possibility of their existence will preclude minimal essentials which take no notice of the structure and laws of mental life.

The determination of what are the independent mental traits, of what are their laws of functioning, and of what adult activities demand them should, for the sake of eugenics, be a prenuptial concern; should, for efficient nurture, be a matter for continual note in the rearing of the individual from the age when his food-getting and other instinctive responses no longer circumscribe his daily life to the close of his formal education; should, for social efficiency, be a determining influence in the choice of a life vocation; and should, in national life, be an intimate issue in establishing comity between nations.

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